# MAT137

- Today: Power series.
- Homework before Thursday's class: watch videos 14.3, 14.4.

# Interval of convergence

Find the interval of convergence of each power series:

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2\, 2^{2n+1}}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{42^n} x^n$

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4. (Hard!) $\displaystyle\sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!}\, x^n$

# What can you conclude?

Think of the power series $\sum_n a_n x^n$. Do not assume $a_n \geq 0$.

In each case, may the given series be absolutely convergent (AC)? conditionally convergent (CC)? divergent (D)? all of them?

| IF | $\sum_n a_n 3^n$ is ... | AC | CC | D |
|---|---|---|---|---|
| THEN | $\sum_n a_n 2^n$ may be ... | ??? | ??? | ??? |
| | $\sum_n a_n (-3)^n$ may be ... | ??? | ??? | ??? |
| | $\sum_n a_n 4^n$ may be ... | ??? | ??? | ??? |

# Writing functions as power series

Using the geometric series, we know how to write the function $F(x) = \dfrac{1}{1-x}$ as a power series centered at 0:

$$F(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

Write these functions as power series centered at 0:

1. $f(x) = \dfrac{1}{1+x}$
2. $g(x) = \dfrac{1}{1-x^2}$
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3. $h(x) = \dfrac{1}{2-x}$
4. $G(x) = \ln(1+x)$

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1. What is the value of the sum $\sum_{n=0}^{\infty} x^n$ ?
2. What is the relation between $\sum_{n}^{\infty} x^n$ and $\sum_{n}^{\infty} nx^{n-1}$ ?
3. Compute the value of the sum $\sum_{n=1}^{\infty} nx^{n-1}$.
4. Compute the value of the sum $\sum_{n=1}^{\infty} nx^n$.
5. Compute the value of the original series