Today: Power series.

Homework before Friday’s class: watch videos 14.3, 14.4.
Find the interval of convergence of each power series:

1. \[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]
2. \[ \sum_{n=1}^{\infty} \frac{(x - 5)^n}{n^2 2^{2n+1}} \]
3. \[ \sum_{n=1}^{\infty} \frac{n^n}{42^n} x^n \]
Interval of convergence

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3. \[ \sum_{n=1}^{\infty} \frac{n^n}{42^n} x^n \]
4. (Hard!) \[ \sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!} x^n \]
What can you conclude?

Think of the power series $\sum_{n} a_n x^n$. Do not assume $a_n \geq 0$.

In each case, may the given series be absolutely convergent (AC)? conditionally convergent (CC)? divergent (D)? all of them?

<table>
<thead>
<tr>
<th>IF</th>
<th>$\sum_{n} a_n 3^n$ is ...</th>
<th>AC</th>
<th>CC</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>THEN</td>
<td>$\sum_{n} a_n 2^n$ may be ...</td>
<td>???</td>
<td>???</td>
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<tr>
<td></td>
<td>$\sum_{n} a_n (-3)^n$ may be ...</td>
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<td></td>
<td>$\sum_{n} a_n 4^n$ may be ...</td>
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</tbody>
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Using the geometric series, we know how to write the function $F(x) = \frac{1}{1 - x}$ as a power series centered at 0:

$$F(x) = \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

Write these functions as power series centered at 0:

1. $f(x) = \frac{1}{1 + x}$
2. $g(x) = \frac{1}{1 - x^2}$
3. $h(x) = \frac{1}{2 - x}$
Writing functions as power series

Using the geometric series, we know how to write the function $F(x) = \frac{1}{1 - x}$ as a power series centered at 0:

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Write these functions as power series centered at 0:

1. $f(x) = \frac{1}{1 + x}$
2. $g(x) = \frac{1}{1 - x^2}$
3. $h(x) = \frac{1}{2 - x}$
4. $G(x) = \ln(1 + x)$
We want to calculate the value of \( \sum_{n=1}^{\infty} \frac{n}{2^n} \)
We want to calculate the value of

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

1. What is the value of the sum

$$\sum_{n=0}^{\infty} x^n$$?

2. What is the relation between

$$\sum_{n} x^n$$ and

$$\sum_{n} nx^{n-1}$$?

3. Compute the value of the sum

$$\sum_{n=1}^{\infty} nx^{n-1}$$.

4. Compute the value of the sum

$$\sum_{n=1}^{\infty} nx^n$$.

5. Compute the value of the original series.