MAT137

- Today: Alternating series.

- Homework before Thursday's class: watch videos 13.15, as well as 13.16, 13.17.

Rapid questions: Convergent or divergent?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

An AST example

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty}(-1)^n\frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

Estimation

Estimate the sum

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

with an error smaller than 0.001. Write your final answer as a rational number (i.e. as a quotient of two integers).

Not quite alternating series

Are these series convergent or divergent?

$$A = 1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \frac{1}{10} - \frac{1}{11} - \frac{1}{12} + \dots$$

$$B = 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9} - \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} - \dots$$

Suggestion: Draw the partial sums on the real line.

A counterexample to the AST?

Construct a series of the form $\sum_{n=1}^{\infty}(-1)^n b_n$ such that

- $b_n > 0$ for all $n \geq 1$
- $\lim_{n\to\infty} b_n = 0$
- the series $\sum_{n=1}^{\infty}(-1)^n b_n$ is divergent.