Today: Integration by substitution.

Homework before Tuesday's class: watch video 9.4, as well as 9.5, 9.6.
Warm up

Calculate

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$$

*Hint*: Use the substitution $u = \sqrt{x}$. 
Use substitutions to compute:

1. $\int e^x \cos(e^x) \, dx$
2. $\int \cot x \, dx$
3. $\int x^2 \sqrt{x+1} \, dx$
4. $\int \frac{e^{2x}}{\sqrt{e^x + 1}} \, dx$
5. $\int \frac{(\ln \ln x)^2}{x \ln x} \, dx$
6. $\int xe^{-x^2} \, dx$
7. $\int e^{-x^2} \, dx$
This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_{0}^{2} \sqrt{x^3 + 1} x^2 \, dx$

Wrong answer

Substitution: $u = x^3 + 1$, $du = 3x^2 \, dx$.

\[
I = \frac{1}{3} \int_{0}^{2} \sqrt{x^3 + 1} \, (3x^2 \, dx) = \frac{1}{3} \int_{0}^{2} u^{1/2} \, du
\]

\[
= \frac{1}{3} \left[ \frac{2}{3} u^{3/2} \right]_{0}^{2} = \frac{1}{9} (x^3 + 1)^{2/3} \bigg|_{0}^{2}
\]

\[
= \frac{52}{9}
\]
Definite integral via substitution

Calculate \( I = \int_{0}^{2} \sqrt{x^3 + 1} \, x^2 \, dx \)

Correct answer #1

Substitution:

\[
\begin{align*}
  u &= x^3 + 1 & \text{When } x = 0, \, u &= 1 \\
  du &= 3x^2 \, dx & \text{When } x = 2, \, u &= 9
\end{align*}
\]

\[
I = \frac{1}{3} \int_{0}^{2} \sqrt{x^3 + 1} \, (3x^2 \, dx) \\
= \frac{1}{3} \int_{1}^{9} u^{1/2} \, du \\
= \frac{2}{9} \left[ u^{3/2} \right]_{1}^{9} = \frac{2}{9} \left( 9^{3/2} - 1^{3/2} \right) = \frac{52}{9}
\]
Definite integral via substitution

Calculate \( I = \int_{0}^{2} \sqrt{x^3 + 1} \, x^2 \, dx \)

Correct answer #2

Substitution:

- First find an antiderivative. Substitution: \( u = x^3 + 1, \quad du = 3x^2 \, dx \).

\[
\int \sqrt{x^3 + 1} \, dx = \frac{1}{3} \int u^{1/2} \, du = \frac{2}{9} u^{3/2} + C = \frac{2}{9} (x^3 + 1)^{3/2} + C
\]

- Then use FTC.

\[
\int_{0}^{2} \sqrt{x^3 + 1} \, dx = \left. \frac{2}{9} (x^3 + 1)^{3/2} \right|_{0}^{2} = \frac{2}{9} (2^3 + 1)^{3/2} - \frac{2}{9} (0 + 1)^{3/2} = \frac{52}{9}
\]
A different kind of substitution

Calculate

\[ \int_{0}^{1} \sqrt{1 - x^2} \, dx \]

using the substitution \( x = \sin \theta \).
Odd functions

**Theorem**

Let $f$ be a continuous function. Let $a > 0$. IF $f$ is odd, THEN

$$\int_{-a}^{a} f(x) \, dx = ???$$

1. Draw a picture to interpret the theorem geometrically.
2. Write down the definition of “odd function”.
3. Prove the theorem!

*Hint:* Write the integral as sum of two pieces. Use a substitution in one of them to show that they cancel with each other.