Today: Integration by substitution.

Homework before Tuesday’s class: watch video 9.4, as well as 9.5, 9.6.
Warm up

Calculate

\[ \int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx \]

*Hint:* Use the substitution \( u = \sqrt{x} \).
Use substitutions to compute:

1. \( \int e^x \cos (e^x) \, dx \)

2. \( \int \cot x \, dx \)

3. \( \int x^2 \sqrt{x + 1} \, dx \)

4. \( \int \frac{e^{2x}}{\sqrt{e^x + 1}} \, dx \)

5. \( \int \frac{(\ln \ln x)^2}{x \ln x} \, dx \)

6. \( \int xe^{-x^2} \, dx \)

7. \( \int e^{-x^2} \, dx \)
Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate \( I = \int_{0}^{2} \sqrt{x^3 + 1} \ x^2 \, dx \)

Wrong answer

Substitution: \( u = x^3 + 1, \ du = 3x^2 \, dx \).

\[
I = \frac{1}{3} \int_{0}^{2} \sqrt{x^3 + 1} \ (3x^2 \, dx) \\
= \frac{1}{3} \frac{2}{3} \left. u^{3/2} \right|_{0}^{2} \\
= \frac{2}{9} (2^3 + 1)^{3/2} - \frac{2}{9} (0 + 1)^{3/2} \\
= \frac{52}{9}
\]

\[
= \frac{1}{3} \int_{0}^{2} u^{1/2} \, du \\
= \frac{1}{9} \left. (x^3 + 1)^{2/3} \right|_{0}^{2} \\
= \frac{52}{9}
\]
Definite integral via substitution

Calculate \( I = \int_0^2 \sqrt{x^3 + 1} \, x^2 \, dx \)

**Correct answer #1**

**Substitution:**

\[
\begin{align*}
  u &= x^3 + 1 \quad \text{When } x = 0, \ u = 1 \\
  du &= 3x^2 \, dx \quad \text{When } x = 2, \ u = 9
\end{align*}
\]

\[
I = \frac{1}{3} \int_0^2 \sqrt{x^3 + 1} \ (3x^2 \, dx)
\]

\[
= \frac{1}{3} \int_1^9 u^{1/2} \, du
\]

\[
= \frac{2}{9} u^{3/2} \bigg|_1^9 = \frac{2}{9} 9^{3/2} - \frac{2}{9} 1^{3/2} = \frac{52}{9}
\]
Calculate \( I = \int_{0}^{2} \sqrt{x^3 + 1} \, x^2 \, dx \)

**Correct answer #2**

Substitution:

- First find an antiderivative. Substitution: \( u = x^3 + 1, \ du = 3x^2 \, dx \).

\[
\int \sqrt{x^3 + 1} \, dx = \frac{1}{3} \int u^{1/2} \, du = \frac{2}{9} u^{3/2} + C = \frac{2}{9} (x^3 + 1)^{3/2} + C
\]

- Then use FTC.

\[
\int_{0}^{2} \sqrt{x^3 + 1} \, dx = \frac{2}{9} (x^3 + 1)^{3/2}\big|_{0}^{2} = \frac{2}{9} (2^3 + 1)^{3/2} - \frac{2}{9} (0 + 1)^{3/2} = \frac{52}{9}
\]
A different kind of substitution

Calculate

$$\int_0^1 \sqrt{1 - x^2} \, dx$$

using the substitution $x = \sin \theta$. 
Odd functions

Theorem

Let \( f \) be a continuous function. Let \( a > 0 \). IF \( f \) is odd, THEN

\[
\int_{-a}^{a} f(x)\,dx = ???
\]

1. Draw a picture to interpret the theorem geometrically.
2. Write down the definition of “odd function”.
3. Prove the theorem!

*Hint*: Write the integral as sum of two pieces. Use a substitution in one of them to show that they cancel with each other.