- Today: Integration by substitution.

- Homework before Tuesday's class: watch video 9.4, as well as 9.5, 9.6.

Warm up

Calculate

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Hint: Use the substitution $u = \sqrt{x}$.

Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int e^x cos\left(e^x\right) dx$
2. $\displaystyle\int \cot x \, dx$
3. $\displaystyle\int x^2\sqrt{x+1} \, dx$
4. $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}} \, dx$
5. $\displaystyle\int \frac{(\ln \ln x)^2}{x \ln x} \, dx$
6. $\displaystyle\int xe^{-x^2} \, dx$
7. $\displaystyle\int e^{-x^2} \, dx$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$
\begin{aligned}
I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2 dx) && = \frac{1}{3}\int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\ \left. u^{3/2}\right|_0^2 && = \frac{1}{9}\left.\left(x^3+1\right)^{2/3}\right|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\,(0+1)^{3/2} && = \frac{52}{9}
\end{aligned}
$$

Definite integral via substitution

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Correct answer #1

Substitution:

$$u = x^3 + 1 \qquad \text{When } x = 0, u = 1$$
$$du = 3x^2 dx \qquad \text{When } x = 2, u = 9$$

$$\begin{aligned} I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2dx) \\ &= \frac{1}{3}\int_{\mathbf{1}}^{\mathbf{9}} u^{1/2}\, du \\ &= \frac{2}{9}\, u^{3/2}\Big|_{\mathbf{1}}^{\mathbf{9}} = \frac{2}{9}9^{3/2} - \frac{2}{9}1^{3/2} = \frac{52}{9} \end{aligned}$$

Definite integral via substitution

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Correct answer #2

Substitution:

- First find an antiderivative. Substitution: $u = x^3 + 1,\ du = 3x^2 dx$.

$$\int \sqrt{x^3+1}\, dx \;=\; \frac{1}{3}\int u^{1/2}\, du \;=\; \frac{2}{9}u^{3/2} + C \;=\; \frac{2}{9}\left(x^3+1\right)^{3/2} + C$$

- Then use FTC.

$$\begin{aligned}\int_0^2 \sqrt{x^3+1}\, dx \;&= \frac{2}{9}\left(x^3+1\right)^{3/2}\Big|_0^2 \\ &= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} \;=\; \frac{52}{9}\end{aligned}$$

A different kind of substitution

Calculate

$$\int_0^1 \sqrt{1 - x^2}\, dx$$

using the substitution $x = \sin\theta$.

Odd functions

Theorem

Let f be a continuous function. Let $a > 0$. IF f is odd, THEN

$$\int_{-a}^{a} f(x)dx = ???$$

1. Draw a picture to interpret the theorem geometrically.
2. Write down the definition of "odd function".
3. Prove the theorem!

   Hint: Write the integral as sum of two pieces. Use a substitution in one of them to show that they cancel with each other.