- Today: FTC Part 1.
- Homework before Wednesday's class: watch videos 8.5, 8.6.
- Test 3 will take place on Friday, February 10, 4-6pm, see details on Quercus.

True, False, or Shrug?

We want to find a function H with domain $\mathbb{R}$ such that $H(1) = -2$ and such that $H'(x) = e^{\sin x}$ for all x. Decide whether each of the following statements is true, false, or we do not have enough information to decide.

1. $\forall C \in \mathbb{R}$, the function $H(x) = \int_0^x e^{\sin t} dt + C$ is a solution.
2. $\exists C \in \mathbb{R}$ s.t. the function $H(x) = \int_0^x e^{\sin t} dt + C$ is a solution.
3. The function $H(x) = \int_0^x e^{\sin t} dt$ is a solution.
4. The function $H(x) = \int_1^x e^{\sin t} dt - 2$ is a solution.
5. There is more than one solution.
6. The function $H(x) = \int_2^x e^{\sin t} dt$ is a solution.

Examples of FTC-1

Compute the derivative of the following functions

1. $F_1(x) = \int_0^1 e^{-t^2} dt.$
2. $F_2(x) = \int_0^x e^{-\sin t} dt.$
3. $F_3(x) = \int_1^{x^2} \frac{\sin t}{t^2} dt.$
4. $F_4(x) = \int_x^7 \sin^3\left(\sqrt{t}\right) dt.$
5. $F_5(x) = \int_{2x}^{x^2} \frac{1}{1+t^3} dt.$

A generalized version of FTC-1

Exercise

Let f, u, v be differentiable functions with domain $\mathbb{R}$. Let us call

$$F(x) = \int_{u(x)}^{v(x)} f(t)dt$$

Find a formula for

$$F'(x)$$

in terms of f, u, v, f', u', v'.

An integral equation

Assume f is a continuous function that satisfies, for every $x \in \mathbb{R}$:

$$\int_0^x e^t f(t)dt = \frac{\sin x}{x^2 + 1}$$

Find an explicit expression for $f(x)$.

An application of FTC-1

Use FTC-1 to prove that, for every $x > 0$

$$\int_0^x \frac{dt}{1+t^2} + \int_0^{1/x} \frac{dt}{1+t^2} = \frac{\pi}{2}$$

What happens for $x < 0$?