
Homework before Wednesday’s class: watch videos 8.5, 8.6, 8.7.

Test 3 will take place on Thursday, February 7, at 6:10-8:00pm.
True, False, or Shrug?

We want to find a function $H$ with domain $\mathbb{R}$ such that $H(1) = -2$ and such that $H'(x) = e^{\sin x}$ for all $x$. Decide whether each of the following statements is true, false, or we do not have enough information to decide.

1. $\forall C \in \mathbb{R}$, the function $H(x) = \int_0^x e^{\sin t} \, dt + C$ is a solution.

2. $\exists C \in \mathbb{R}$ s.t. the function $H(x) = \int_0^x e^{\sin t} \, dt + C$ is a solution.

3. The function $H(x) = \int_0^x e^{\sin t} \, dt$ is a solution.

4. The function $H(x) = \int_1^x e^{\sin t} \, dt - 2$ is a solution.

5. There is more than one solution.

6. The function $H(x) = \int_2^x e^{\sin t} \, dt$ is a solution.
Examples of FTC-1

Compute the derivative of the following functions

1. $F_1(x) = \int_0^1 e^{-t^2} \, dt$.  
2. $F_2(x) = \int_0^x e^{-\sin t} \, dt$.  
3. $F_3(x) = \int_1^{x^2} \frac{\sin t}{t^2} \, dt$.  
4. $F_4(x) = \int_x^7 \sin^3(\sqrt{t}) \, dt$.  
5. $F_5(x) = \int_{2x}^{x^2} \frac{1}{1 + t^3} \, dt$.  

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A generalized version of FTC-1

Exercise

Let $f$, $u$, $v$ be differentiable functions with domain $\mathbb{R}$. Let us call

$$F(x) = \int_{u(x)}^{v(x)} f(t) dt$$

Find a formula for

$$F'(x)$$

in terms of $f$, $u$, $v$, $f'$, $u'$, $v'$. 
Assume $f$ is a continuous function that satisfies, for every $x \in \mathbb{R}$:

$$\int_0^x e^t f(t) dt = \frac{\sin x}{x^2 + 1}$$

Find an explicit expression for $f(x)$. 
An application of FTC-1

Use FTC-1 to prove that, for every $x > 0$

$$\int_0^x \frac{dt}{1 + t^2} + \int_0^{1/x} \frac{dt}{1 + t^2} = \frac{\pi}{2}$$

What happens for $x < 0$?