MAT137

- Today: Antiderivatives and indefinite integrals.
- Homework before Tuesday's class: watch videos 8.3, 8.4.
- Test 3 will take place on Friday, February 10, 4-6pm, see details on Quercus.

Initial Value Problem

Find a function f such that

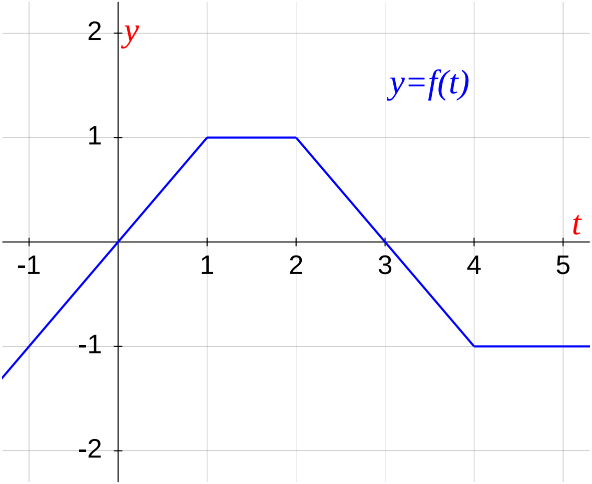
- For every $x \in \mathbb{R}$, $f''(x) = \sin x + x^2$,
- $f'(0) = 5$,
- $f(0) = 7$.

Functions defined by integrals

Which ones of these are valid ways to define functions?

1. $F(x) = \int_0^x \frac{t}{1+t^8}\,dt$
2. $F(x) = \int_0^x \frac{x}{1+x^8}\,dx$
3. $F(x) = \int_0^x \frac{x}{1+t^8}\,dt$
4. $F(x) = \int_0^{x^2} \frac{t}{1+t^8}\,dt$
5. $F(x) = \int_{\sin x}^{e^x} \frac{t}{1+t^8}\,dt$
6. $F(x) = \int_0^3 \frac{t}{1+x^2+t^8}\,dt$
7. $F(x) = x\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$
8. $F(x) = t\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$

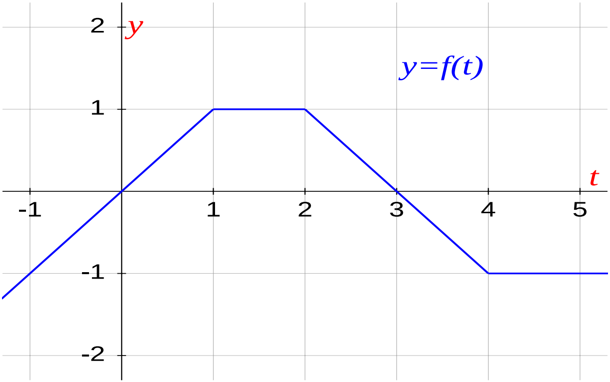
Towards FTC



Compute:

1. $\int_0^1 f(t)dt$
2. $\int_0^2 f(t)dt$
3. $\int_0^3 f(t)dt$
4. $\int_0^4 f(t)dt$
5. $\int_0^5 f(t)dt$

Towards FTC (continued)



Call $F(x) = \int_0^x f(t)dt$. This is a new function.

- Sketch the graph of $y = F(x)$.
- Using the graph you just sketched, sketch the graph of $y = F'(x)$.

Antiderivatives

Compute these antiderivatives by guess-and-check.

1. $\int x^5 dx$
2. $\int \left(3x^8 - 18x^5 + x + 1\right) dx$
3. $\int \sqrt[3]{x}\ dx$
4. $\int \frac{1}{x^9}\ dx$
5. $\int \sqrt{x}\left(x^2 + 5\right) dx$
6. $\int \frac{1}{e^{2x}}\ dx$
7. $\int \sin(3x)\ dx$
8. $\int \cos(3x + 2)\ dx$
9. $\int \sec^2 x\ dx$
10. $\int \sec x \tan x\ dx$
11. $\int \frac{1}{x}\ dx$
12. $\int \frac{1}{x + 3}\ dx$

Trig-exp antiderivatives

1. Calculate

$$\frac{d}{dx}\left[e^x \sin x\right], \qquad \frac{d}{dx}\left[e^x \cos x\right].$$

2. Use the previous answer to calculate

$$\int e^x \sin x \; dx, \qquad \int e^x \cos x \; dx.$$

A harder antiderivative

1. Calculate

$$\frac{d}{dx}\left[\arctan x\right], \qquad \frac{d}{dx}\left[\frac{x}{1+x^2}\right].$$

2. Use the previous answer to calculate

$$\int \frac{1}{\left(1+x^2\right)^2}\,dx$$