Today: Integrable functions.

Homework before Tuesday’s class: watch videos 7.7, 7.8, 7.11.
Warm up: partitions

Which ones are partitions of $[0, 2]$?

1. $[0, 2]$
2. $(0, 2)$
3. $\{0, 2\}$
4. $\{1, 2\}$
5. $\{0, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$
Partitions of different intervals

1. Let $P$ be a partition of $[0, 1]$. Let $Q$ be a partition of $[1, 2]$. How do we construct a partition of $[0, 2]$ from them?

2. Let $R$ be a partition of $[0, 2]$. How do we construct partitions of $[0, 1]$ and $[1, 2]$ from it?
Let \( f(x) = \cos x \).
Consider the partition \( P = \{0, 1, 2, 4\} \) of the interval \([0, 4]\).
Calculate \( L_P(f) \) and \( U_P(f) \).
Lower and upper sums

Let \( f \) be a decreasing, bounded function on \([a, b]\). Let \( P = \{x_0, x_1, \ldots, x_N\} \) be a partition of \([a, b]\).

Which one (or ones) is a valid equation for \( L_P(f) \)? For \( U_f(P) \)?

1. \[ \sum_{i=0}^{N} \Delta x_i f(x_i) \]
2. \[ \sum_{i=1}^{N} \Delta x_i f(x_i) \]
3. \[ \sum_{i=0}^{N-1} \Delta x_i f(x_i) \]
4. \[ \sum_{i=1}^{N} \Delta x_i f(x_{i+1}) \]
5. \[ \sum_{i=1}^{N} \Delta x_i f(x_{i-1}) \]
6. \[ \sum_{i=0}^{N-1} \Delta x_{i+1} f(x_{i}) \]

Recall: \( \Delta x_i = x_i - x_{i-1} \).
Assume

\[
L_P(f) = 2, \quad U_P(f) = 6 \\
L_Q(f) = 3, \quad U_Q(f) = 8
\]

1. Is \( P \subseteq Q \)?
2. Is \( Q \subseteq P \)?
3. What can you say about \( L_{P \cup Q}(f) \) and \( U_{P \cup Q}(f) \)?
1. Let $f$ be a bounded function on $[0, 1]$. Assume $f$ is not constant. Prove that there exists a partition $P$ of $[0, 1]$ such that $L_P(f) \neq U_P(f)$.

*Hint:* This is easier than it looks.
1. Let $f$ be a bounded function on $[0, 1]$. Assume $f$ is not constant. Prove that there exists a partition $P$ of $[0, 1]$ such that $L_P(f) \neq U_P(f)$.

*Hint:* This is easier than it looks.

2. For which functions $f$ is there a partition $P$ of $[0, 1]$ such that $L_P(f) = U_P(f)$?