- Today: Integrable functions.

- Homework before Tuesday's class: watch videos 7.7, 7.8, 7.11.

Warm up: partitions

Which ones are partitions of $[0, 2]$?

1. $[0, 2]$
2. $(0, 2)$
3. $\{0, 2\}$
4. $\{1, 2\}$
5. $\{0, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$

Partitions of different intervals

1. Let P be a partition of $[0, 1]$.
   Let Q be a partition of $[1, 2]$.
   How do we construct a partition of $[0, 2]$ from them?

2. Let R be a partition of $[0, 2]$.
   How do we construct partitions of $[0, 1]$ and $[1, 2]$ from it?

Warm up: lower and upper sums

Let $f(x) = \cos x$.
Consider the partition $P = \{0, 1, 2, 4\}$ of the interval $[0, 4]$.
Calculate $L_P(f)$ and $U_P(f)$.

Lower and upper sums

Let f be a **decreasing**, bounded function on $[a, b]$.
Let $P = \{x_0, x_1, \ldots, x_N\}$ be a partition of $[a, b]$

Which one (or ones) is a valid equation for $L_P(f)$? For $U_f(P)$?

1. $\displaystyle\sum_{i=0}^{N} \Delta x_i\, f(x_i)$
2. $\displaystyle\sum_{i=1}^{N} \Delta x_i\, f(x_i)$
3. $\displaystyle\sum_{i=0}^{N-1} \Delta x_i\, f(x_i)$
4. $\displaystyle\sum_{i=1}^{N} \Delta x_i\, f(x_{i+1})$
5. $\displaystyle\sum_{i=1}^{N} \Delta x_i\, f(x_{i-1})$
6. $\displaystyle\sum_{i=0}^{N-1} \Delta x_{i+1}\, f(x_i)$

Recall: $\Delta x_i = x_i - x_{i-1}$.

Joining partitions

Assume

$$L_P(f) = 2, \quad U_P(f) = 6$$
$$L_Q(f) = 3, \quad U_Q(f) = 8$$

1. Is $P \subseteq Q$?
2. Is $Q \subseteq P$?
3. What can you say about $L_{P \cup Q}(f)$ and $U_{P \cup Q}(f)$?

1. Let f be a bounded function on $[0, 1]$.
   Assume f is not constant.
   Prove that there exists a partition P of $[0, 1]$ such that

$$L_P(f) \neq U_P(f).$$

   Hint: This is easier than it looks.

1. Let f be a bounded function on $[0, 1]$.
   Assume f is not constant.
   Prove that there exists a partition P of $[0, 1]$ such that

$$L_P(f) \neq U_P(f).$$

   Hint: This is easier than it looks.

2. For which functions f
   is there a partition P of $[0, 1]$ such that
   $L_P(f) = U_P(f)$?