Today: Integrable functions.

Homework before Monday’s class: watch (again) videos 7.5, 7.6, 7.7, 7.8, 7.9.
Which ones are partitions of $[0, 2]$?

1. $[0, 2]$
2. $(0, 2)$
3. $\{0, 2\}$
4. $\{1, 2\}$
5. $\{0, 1.5, 1.6, 1.7, 1.8, 1.9, 2\}$
1. Let $P$ be a partition of $[0, 1]$. Let $Q$ be a partition of $[1, 2]$. How do we construct a partition of $[0, 2]$ from them?

2. Let $R$ be a partition of $[0, 2]$. How do we construct partitions of $[0, 1]$ and $[1, 2]$ from it?
Let $f(x) = \cos x$.  
Consider the partition $P = \{0, 1, 2, 4\}$ of the interval $[0, 4]$.
Calculate $L_P(f)$ and $U_P(f)$. 
Lower and upper sums

Let $f$ be a decreasing, bounded function on $[a, b]$. Let $P = \{x_0, x_1, \ldots, x_N\}$ be a partition of $[a, b]$

Which one (or ones) is a valid equation for $L_P(f)$? For $U_f(P)$?

1. $\sum_{i=0}^{N} \Delta x_i f(x_i)$
2. $\sum_{i=1}^{N} \Delta x_i f(x_i)$
3. $\sum_{i=0}^{N-1} \Delta x_i f(x_i)$
4. $\sum_{i=1}^{N} \Delta x_i f(x_{i+1})$
5. $\sum_{i=1}^{N} \Delta x_i f(x_{i-1})$
6. $\sum_{i=0}^{N-1} \Delta x_{i+1} f(x_i)$

Recall: $\Delta x_i = x_i - x_{i-1}$. 
Joining partitions

Assume

\[ L_P(f) = 2, \quad U_P(f) = 6 \]
\[ L_Q(f) = 3, \quad U_Q(f) = 8 \]

1. Is \( P \subseteq Q? \)
2. Is \( Q \subseteq P? \)
3. What can you say about \( L_{P \cup Q}(f) \) and \( U_{P \cup Q}(f) \)?
More on upper/lower sums

1. Let $f$ be a bounded function on $[0, 1]$. Assume $f$ is not constant. Prove that there exists a partition $P$ of $[0, 1]$ such that

$$L_P(f) \neq U_P(f).$$

*Hint:* This is easier than it looks.
1. Let $f$ be a bounded function on $[0, 1]$. Assume $f$ is not constant. Prove that there exists a partition $P$ of $[0, 1]$ such that $L_P(f) \neq U_P(f)$.

*Hint:* This is easier than it looks.

2. For which functions $f$ is there a partition $P$ of $[0, 1]$ such that $L_P(f) = U_P(f)$?