- Today: Sums and sigmas.
- Homework before Wednesday's class: watch videos 7.3, 7.4.
- Homework before Thursday's class: watch videos 7.5, 7.6.

Warm-up: sums

Recall:

$$\sum_{i=1}^{n} a_i \;=\; a_1 + a_2 + a_3 + \ldots + a_n$$

Compute

1. $\sum_{i=2}^{4}(2i+1)$
2. $\sum_{i=2}^{4} 2i+1$
3. $\sum_{j=2}^{4}(2i+1)$

Write these sums with sigma notation

1. $1^5 + 2^5 + 3^5 + 4^5 + \ldots + 100^5$

2. $\frac{2}{4^2} + \frac{2}{5^2} + \frac{2}{6^2} + \frac{2}{7^2} + \ldots + \frac{2}{N^2}$

3. $\cos 0 - \cos 1 + \cos 2 - \cos 3 + \ldots \pm \cos(N+1)$

4. $\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots + \frac{1}{(2N)!}$

5. $\frac{1}{1!} - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots + \frac{1}{81!}$

6. $\frac{x^2}{3!} + \frac{2x^3}{4!} + \frac{3x^4}{5!} + \frac{4x^5}{6!} + \ldots + \frac{999x^{1000}}{1001!}$

Double sums

Compute:

1. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{N} 1$
2. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} 1$
3. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} i$
4. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} k$
5. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i} (ik)$

Useful formulas:

$$\sum_{j=1}^{N} j = \frac{N(N+1)}{2}, \quad \sum_{j=1}^{N} j^2 = \frac{N(N+1)(2N+1)}{6}, \quad \sum_{j=1}^{N} j^3 = \frac{N^2(N+1)^2}{4}$$

Re-writing sums

1. $$\sum_{i=1}^{100} \tan i - \sum_{i=1}^{50} \tan i = \sum_{???}^{???} ???$$

2. $$\sum_{i=1}^{N} (2i-1)^5 = \sum_{i=0}^{N-1} ???$$

3. $$\left[\sum_{k=1}^{N} x^k\right] + \left[\sum_{k=0}^{N} k\, x^{k+1}\right] = \left[\sum_{k=???}^{???} ???x^k\right] + ???$$

Telescopic sum

Calculate the exact value of

$$\sum_{i=1}^{2{,}022} \left[\frac{1}{i} - \frac{1}{i+1} \right]$$

Hint: Write down the first few terms.

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Hint: Write down the first few terms.

Calculate the exact value of

$$\sum_{i=1}^{10,000} \frac{1}{i(i+1)}$$

Fubini-Tonelli

- $A_{i,k}$ is a function of 2 variables.
  For example, $A_{i,k} = \dfrac{i}{k+i^2}$.
- Decide what to write instead of each "?" so that the following identity is true:

$$\sum_{i=1}^{N} \sum_{k=1}^{i} A_{i,k} = \sum_{k=?}^{?} \sum_{i=?}^{?} A_{i,k}$$