MAT137

- Today: Volumes (cont'd)

- Tomorrow, on Thursday we will start sequences. Please watch videos 11.1, 11.2. I strongly recommend that you watch these videos!

An equation for volumes by "cylindrical shells"

Let $a < b$.
Let f be a continuous, positive function defined on $[a, b]$.
Let R be the region in the first quadrant bounded between the graph of f and the x-axis.
Find a formula for the volume of the solid of revolution obtained by rotation the region R around the y-axis.

Volume of a sphere

Last time we found the volume of a sphere by the cross-section method, by rotating a circle around the x-axis.

Recall how you did this. Recall the equation of a circle (or half-circle) of radius R centered at $(0, 0)$.

Now reorient your half-circle, rotate it about the y-axis and find the sphere volume using the shell method.

Volume of a circus tent

A circus tent is made by taking the graph of
$y = (x-1)^4 + 1,\ 0 \le x \le 1$ and rotating it around the y-axis.

Find the volume by cutting it into cylindrical shells.

Now try to find the volume by the cross-section method, aka the "washer method."

Imagine that we had a more complicated function. Which method would work better?

Challenge

Two cylinders have the same radius R and their axes meet at a right angle. Find the volume of their intersection.

Hint: You can slice the resulting solid by parallel cuts in three different directions. One of the three makes the problem much, much simpler than the other two.