## MAT137

- Today: Volumes
- Homework before Wednesday's class: watch video 10.2.
- Term test 3: Friday, February 10, 4-6pm.


## Volume of a cylinder

Find the volume of a cylinder with base radius 1 and height 2 .
First step: put this cylinder in a convenient place in our usual coordinate system.

Second step: find the cross-sectional area.
Third step: express the volume as an integral.

## An equation for volumes by "slicing"

Let $a<b$.
Let $f$ be a continuous, positive function defined on $[a, b]$. Let $R$ be the region in the first quadrant bounded between the graph of $f$ and the $x$-axis.
Find a formula for the volume of the solid of revolution obtained by rotation the region $R$ around the $x$-axis.

## Sphere

You know a formula for the volume of a sphere with radius $R$. Now you are able to prove it!

1. Write an equation for the circle with radius $R$ centered at $(0,0)$.
2. If you rotate this circle around the $x$-axis, it will produce a sphere. Compute its volume as an integral by slicing it like a carrot.

## Ball $=$ cylinder - cone (Archimedes, 3rd century BC)



Cavalieri's Principle: "equal sections $\Rightarrow$ equal volumes"
Compare areas of sections at height $y$ and compute volumes as integrals:


## Many axes of rotation

Let $R$ be the region between the graphs of $y=x$ and $y=x^{2}$ is rotated about the $x$-axis.

What does the cross-section look like?
Find the volume.
What about if we rotate about the line $y=-1$ ?
What happens when we rotate it about the $y$-axis?

