Today: Volumes (cont’d)

On Friday we will start sequences. Please watch videos 11.1, 11.2, 11.3. I strongly recommend that you watch these videos!
An equation for volumes by “cylindrical shells”

Let \( a < b \).
Let \( f \) be a continuous, positive function defined on \([a, b]\).
Let \( R \) be the region in the first quadrant bounded between the graph of \( f \) and the \( x \)-axis.
Find a formula for the volume of the solid of revolution obtained by rotation the region \( R \) around the \( y \)-axis.
Volume of a sphere

Last time we found the volume of a sphere by the cross-section method, by rotating a circle around the x-axis.

Recall how you did this. Recall the equation of a circle (or half-circle) of radius $R$ centered at $(0, 0)$.

Now reorient your half-circle, rotate it about the y-axis and find the sphere volume using the shell method.
A circus tent is made by taking the graph of 
\[ y = (x - 1)^4 + 1, \ 0 \leq x \leq 1 \] and rotating it around the y-axis.

Find the volume by cutting it into cylindrical shells.

Now try to find the volume by the cross-section method, aka the “washer method.”

Imagine that we had a more complicated function. Which method would work better?
Two cylinders have the same radius $R$ and their axes meet at a right angle. Find the volume of their intersection.

*Hint:* You can slice the resulting solid by parallel cuts in three different directions. One of the three makes the problem much, much simpler than the other two.