

- Today: Integration of rational functions.
- Homework before Tuesday's class: watch video 10.1.

Rational integrals

1. Calculate $\int \frac{1}{x+a} dx$

2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x+3}$

3. Calculate $\int \frac{-x+6}{x^2+3x} dx$

4. Calculate $\int \frac{1}{x^2+3x} dx$

5. Calculate $\int \frac{1}{x^3-x} dx$

The integral of secant

Compute

$$\int \sec x \, dx$$

using the substitution $u = \sin x$.

Repeated factors

1. Calculate $\int \frac{1}{(x+1)^n} dx$ for $n > 1$
2. Calculate $\int \frac{(x+1) - 1}{(x+1)^2} dx$
3. Calculate $\int \frac{2x+6}{(x+1)^2} dx$
4. Calculate $\int \frac{x^2}{(x+1)^3} dx$
5. How would you calculate $\int \frac{\text{polynomial}}{(x+1)^3} dx$?

Irreducible quadratics

1. Calculate $\int \frac{1}{x^2 + 1} dx$ and $\int \frac{x}{x^2 + 1} dx$.

Hint: These two are very short.

2. Calculate $\int \frac{2x + 3}{x^2 + 1} dx$

3. Calculate $\int \frac{x^2}{x^2 + 1} dx$

4. Calculate $\int \frac{x}{x^2 + x + 1} dx$

Hint: Transform it into one like the previous ones

1. How could we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)} dx ?$$

1. How could we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)} dx ?$$

2. How could we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)x^4(x^2+1)(x^2+4x+7)} dx ?$$

A harder antiderivative

1. Calculate

$$\frac{d}{dx} [\arctan x], \quad \frac{d}{dx} \left[\frac{x}{1+x^2} \right].$$

2. Use the previous answer to calculate

$$\int \frac{1}{(1+x^2)^2} dx$$

3. Calculate

$$\int \frac{1}{(1+x^2)^3} dx$$