Today: Integration of trigonometric functions.

Homework before Thursday’s class: watch videos 9.10, as well as 9.11, 9.12.
We want to compute the number

\[ A = \int_{-\pi/2}^{3\pi} \sin^2 x \, dx. \]
Warm up: Symmetry

We want to compute the number

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Sketch the graph of \( y = \sin^2 x \). Look at it and search for symmetries. You can get the answer without performing any integration.

Check it using \( \sin^2 x = \frac{1 - \cos(2x)}{2} \)
Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

1. \( \int \sin^{10} x \cos x \, dx \)
2. \( \int \sin^{10} x \cos^3 x \, dx \)
3. \( \int e^{\cos x} \cos x \sin^5 x \, dx \)
4. \( \int \cos^2 x \, dx \)
5. \( \int \sin^4 x \, dx \)
6. \( \int \csc x \, dx \)

Useful trig identities

\[
\sin^2 x + \cos^2 x = 1 \\
\tan^2 x + 1 = \sec^2 x \\
\sin^2 x = \frac{1 - \cos(2x)}{2} \\
\cos^2 x = \frac{1 + \cos(2x)}{2}
\]
A reduction formula

Let $I_n = \int_0^{2\pi} \sin^n x \, dx$.

1. Compute $I_0$ and $I_1$.

2. Starting with $I_n$, use integration by parts. Then use the main trig identity to obtain an equation involving $I_n$ and $I_{n-2}$.

3. Use the previous answers to get a formula for $I_n$ for every positive integer $n$.

4. Compute $I_8$. (The answer should be $\frac{35}{64} \pi$.)
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Integral of products of sin and cos

We want to compute

\[ I = \int \sin^3 x \cos^2 x \, dx \]

1. Attempt the substitution \( u = \sin x \)
2. Attempt the substitution \( u = \cos x \)
3. One worked better than the other. Which one? Why?
   Finish the problem.
Integral of products of sin and cos

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\[ I = \int \sin^3 x \cos^2 x \, dx \]

1. Attempt the substitution \( u = \sin x \)
2. Attempt the substitution \( u = \cos x \)
3. One worked better than the other. Which one? Why?
   Finish the problem.
4. Assume we want to compute

\[ \int \sin^n x \cos^m x \, dx \]

When will the substitution \( u = \sin x \) be helpful?
When will the substitution \( u = \cos x \) be helpful?
Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $???$, then try the substitution $u = \tan x$.
- If $???$, then try the substitution $u = \sec x$.

**Hint:** You will need

$$\frac{d}{dx} [\tan x] = \ldots$$

$$\frac{d}{dx} [\sec x] = \ldots$$

- The trig identity involving sec and tan