- Today: Improper integrals.

- Homework before Wednesday's class: watch videos 12.7, 12.8.

Recall the definitions

1. Let f be a bounded, continuous function on $[c, \infty)$. How do we define the improper integral
$$\int_c^\infty f(x)dx\,?$$
2. Let f be a continuous function on $(a, b]$. How do we define the improper integral
$$\int_a^b f(x)dx\,?$$

Calculate, using the definition of improper integral

$$\int_1^{\infty} \frac{1}{x^2 + x} dx$$

Hint: $\dfrac{1}{x^2 + x} = \dfrac{(x+1) - (x)}{x(x+1)}$

The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\displaystyle\int_1^{\infty} \frac{1}{x^p}\, dx$
2. $\displaystyle\int_0^{1} \frac{1}{x^p}\, dx$
3. $\displaystyle\int_0^{\infty} \frac{1}{x^p}\, dx$

A "simple" integral

What is $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx$?

A “simple” integral

What is $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx$?

1. $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx = (\ln|x|)\Big|_{-1}^{1} = \ln|1| - \ln|-1| = 0$
2. $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx = 0$ because $f(x) = \dfrac{1}{x}$ is an odd function.
3. $\displaystyle\int_{-1}^{1} \frac{1}{x}\, dx$ is divergent.

Collection of antiderivatives

Let f be a positive, continuous function with domain $\mathbb{R}$.
We know two ways to describe a collection of antiderivatives:

1. $G(x) + C$ for $C \in \mathbb{R}$, where G is any one antiderivative.
2. The collection of functions F_a for $a \in \mathbb{R}$, where

$$F_a(x) = \int_a^x f(t)dt$$

These two collections are not always the same.

- Find one function f for which they are the same.
- Find one function f for which they are not the same.
- In general, when are they the same?

Hint: https://tinyurl.com/137antiderivatives