Today: Improper integrals.

Homework before Wednesday’s class: watch videos 12.7, 12.8.
Recall the definitions

1. Let $f$ be a bounded, continuous function on $[c, \infty)$. How do we define the improper integral

$$\int_c^\infty f(x) \, dx$$

2. Let $f$ be a continuous function on $(a, b]$. How do we define the improper integral

$$\int_a^b f(x) \, dx$$
Calculate, using the definition of improper integral

\[ \int_{1}^{\infty} \frac{1}{x^2 + x} \, dx \]

**Hint:**

\[ \frac{1}{x^2 + x} = \frac{(x + 1) - x}{x(x + 1)} \]
The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\int_{1}^{\infty} \frac{1}{x^p} \, dx$

2. $\int_{0}^{1} \frac{1}{x^p} \, dx$

3. $\int_{0}^{\infty} \frac{1}{x^p} \, dx$
A “simple” integral

What is \( \int_{-1}^{1} \frac{1}{x} \, dx \)?
A “simple” integral

What is \( \int_{-1}^{1} \frac{1}{x} \, dx \) ?

1. \( \int_{-1}^{1} \frac{1}{x} \, dx = (\ln |x|) \bigg|_{-1}^{1} = \ln |1| - \ln |-1| = 0 \)

2. \( \int_{-1}^{1} \frac{1}{x} \, dx = 0 \) because \( f(x) = \frac{1}{x} \) is an odd function.

3. \( \int_{-1}^{1} \frac{1}{x} \, dx \) is divergent.
Collection of antiderivatives

Let \( f \) be a positive, continuous function with domain \( \mathbb{R} \). We know two ways to describe a collection of antiderivatives:

1. \( G(x) + C \) for \( C \in \mathbb{R} \), where \( G \) is any one antiderivative.
2. The collection of functions \( F_a \) for \( a \in \mathbb{R} \), where

\[
F_a(x) = \int_a^x f(t) \, dt
\]

These two collections are not always the same.

- Find one function \( f \) for which they are the same.
- Find one function \( f \) for which they are not the same.
- In general, when are they the same?

Hint: [https://tinyurl.com/137antiderivatives](https://tinyurl.com/137antiderivatives)