MAT137

- Today: Integration of trigonometric functions.

- Homework before Thursday's class: watch videos 9.10, as well as 9.11, 9.12.

Warm up: Symmetry

We want to compute the number

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Sketch the graph of $y = \sin^2 x$. Look at it and search for symmetries. **You can get the answer without performing any integration.**

Check it using $\sin^2 x = \frac{1-\cos(2x)}{2}$

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

1. $\int \sin^{10} x \cos x \; dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int e^{\cos x} \cos x \sin^5 x \, dx$
4. $\int \cos^2 x \, dx$
5. $\int \sin^4 x \, dx$
6. $\int \csc x \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

A reduction formula

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   Then use the main trig identity to obtain an equation involving I_n and I_{n-2}.
3. Use the previous answers to get a formula for I_n for every positive integer I_n.
4. Compute I_8. (The answer should be $\frac{35}{64}\pi$).

Integral of products of sin and cos

We want to compute

$$I = \int \sin^3 x \cos^2 x \, dx$$

1. Attempt the substitution $u = \sin x$
2. Attempt the substitution $u = \cos x$
3. One worked better than the other. Which one? Why? Finish the problem.

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4. Assume we want to compute

$$\int \sin^n x \cos^m x \, dx$$

When will the substitution $u = \sin x$ be helpful?
When will the substitution $u = \cos x$ be helpful?

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

Hint: You will need

- $\dfrac{d}{dx}[\tan x] = \ldots$
- $\dfrac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan