Today: Integration of trigonometric functions.

Homework before Thursday’s class: watch videos 9.10, as well as 9.11, 9.12.
We want to compute the number

\[ A = \int_{-\pi/2}^{3\pi} \sin^2 x \, dx. \]
Warm up: Symmetry

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Sketch the graph of \( y = \sin^2 x \). Look at it and search for symmetries. **You can get the answer without performing any integration.**

Check it using \( \sin^2 x = \frac{1-\cos(2x)}{2} \)
Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

1. $\int \sin^{10} x \cos x \, dx$
2. $\int \sin^{10} x \cos^{3} x \, dx$
3. $\int e^{\cos x} \cos x \sin^{5} x \, dx$
4. $\int \cos^{2} x \, dx$
5. $\int \sin^{4} x \, dx$
6. $\int \csc x \, dx$

Useful trig identities

$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x = \frac{1 - \cos(2x)}{2}$
$\cos^2 x = \frac{1 + \cos(2x)}{2}$
A reduction formula

Let \( I_n = \int_0^{2\pi} \sin^n x \, dx \).

1. Compute \( I_0 \) and \( I_1 \).

2. Starting with \( I_n \), use integration by parts. Then use the main trig identity to obtain an equation involving \( I_n \) and \( I_{n-2} \).

3. Use the previous answers to get a formula for \( I_n \) for every positive integer.

4. Compute \( I_8 \). The answer should be \( \frac{35}{64} \pi \).
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We want to compute

\[ I = \int \sin^3 x \cos^2 x \, dx \]

1. Attempt the substitution \( u = \sin x \)
2. Attempt the substitution \( u = \cos x \)
3. One worked better than the other. Which one? Why?

Finish the problem.
We want to compute

\[ I = \int \sin^3 x \cos^2 x \, dx \]

1. Attempt the substitution \( u = \sin x \)
2. Attempt the substitution \( u = \cos x \)
3. One worked better than the other. Which one? Why?

Finish the problem.

4. Assume we want to compute

\[ \int \sin^n x \cos^m x \, dx \]

When will the substitution \( u = \sin x \) be helpful?
When will the substitution \( u = \cos x \) be helpful?
To integrate \[ \int \sec^n x \tan^m x \, dx \]

- If \( n = 1 \), then try the substitution \( u = \tan x \).
- If \( m = 1 \), then try the substitution \( u = \sec x \).

**Hint:** You will need

- \( \frac{d}{dx} \left[ \tan x \right] = \ldots \)
- \( \frac{d}{dx} \left[ \sec x \right] = \ldots \)

- The trig identity involving sec and tan