Today: Integration of rational functions.

Homework before Monday’s class: watch video 10.1.
Rational integrals

1. Calculate $\int \frac{1}{x + a} \, dx$

2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x + 3}$

3. Calculate $\int \frac{-x + 6}{x^2 + 3x} \, dx$

4. Calculate $\int \frac{1}{x^2 + 3x} \, dx$

5. Calculate $\int \frac{1}{x^3 - x} \, dx$
Compute \[ \int \sec x \, dx \]

using the substitution \( u = \sin x \).
Repeated factors

1. Calculate \( \int \frac{1}{(x + 1)^n} \, dx \) for \( n > 1 \)

2. Calculate \( \int \frac{(x + 1) - 1}{(x + 1)^2} \, dx \)

3. Calculate \( \int \frac{2x + 6}{(x + 1)^2} \, dx \)

4. Calculate \( \int \frac{x^2}{(x + 1)^3} \, dx \)

5. How would you calculate \( \int \frac{\text{polynomial}}{(x + 1)^3} \, dx \)?
Irreducible quadratics

1. Calculate \( \int \frac{1}{x^2 + 1} \, dx \) and \( \int \frac{x}{x^2 + 1} \, dx \).
   
   *Hint:* These two are very short.

2. Calculate \( \int \frac{2x + 3}{x^2 + 1} \, dx \)

3. Calculate \( \int \frac{x^2}{x^2 + 1} \, dx \)

4. Calculate \( \int \frac{x}{x^2 + x + 1} \, dx \)

   *Hint:* Transform it into one like the previous ones
1. How could we compute an integral of the form

\[ \int \frac{\text{polynomial}}{(x + 1)^3(x + 2)} \, dx \]
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\[
\int \frac{\text{polynomial}}{(x + 1)^3(x + 2)} \, dx
\]

2. How could we compute an integral of the form
\[
\int \frac{\text{polynomial}}{(x + 1)^3(x + 2)x^4(x^2 + 1)(x^2 + 4x + 7)} \, dx
\]
A harder antiderivative

1. Calculate
\[
\frac{d}{dx} \left[ \arctan x \right], \quad \frac{d}{dx} \left[ \frac{x}{1 + x^2} \right].
\]

2. Use the previous answer to calculate
\[
\int \frac{1}{(1 + x^2)^2} \, dx.
\]
A harder antiderivative

1. Calculate

\[ \frac{d}{dx} \left[ \arctan x \right], \quad \frac{d}{dx} \left[ \frac{x}{1 + x^2} \right]. \]

2. Use the previous answer to calculate

\[ \int \frac{1}{(1 + x^2)^2} \, dx \]

3. Calculate

\[ \int \frac{1}{(1 + x^2)^3} \, dx \]