- Today: The Big Theorem.

- Homework for the Reading Week: watch videos 12.1, 12.4, 12.5, as well as 12.2, 12.3, 12.6.

Calculations

1. $$\lim_{n\to\infty} \frac{n! + 2e^n}{3n! + 4e^n}$$

2. $$\lim_{n\to\infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$$

3. $$\lim_{n\to\infty} \frac{5n^5 + 5^n + 5n!}{n^n}$$

Big Theorem

If we have two sequences $\{a_n\}$ and $\{b_n\}$, then we write $a_n << b_n$ if

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0\,.$$

Order the following sequences according to $<<$:

$$n^2,\ n^3+n,\ 46n+7,\ n^{3^n},\ n\ln n,\ n^{1.2},\ n^{n!},\ 4^n n^3,\ n^{2n+7}$$

Useful: Stirling's formula

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \qquad \text{as} \quad n \to \infty$$

Use this formula to construct two sequences $\{u_n\}_n, \{v_n\}_n, \{x_n\}_n, \{y_n\}_n$, such that they all diverge to infinity, and for every $a > 0$ and for every $c > 1$:

$$u_n \;<<\; \ln n \;<<\; v_n \;<<\; n^a \;<<$$
$$<< c^n \;<<\; x_n \;<<\; n! \;<<\; y_n \;<<\; n^n$$

Review: a recursive sequence

$$a_1 = 1,\ a_{n+1} = \sqrt{2a_n + 3}$$

The first few terms are

$$1,\ 2.23...,\ 2.73...,\ 2.90...$$

Assume that this sequence converges. Prove that the limit is 3.

Use the monotone convergence theorem to prove that our sequence converges.

What would happen if we used $a_1 = 4,\ a_{n+1} = \sqrt{2a_n + 3}$?