MAT137

- Today: Theorems about sequences.

- Homework before Thursday's class: watch videos 11.7, 11.8.

Review – True or False

1. If a sequence is convergent, then it is bounded above.
2. If a sequence is convergent, then it is eventually monotonic.
3. If a sequence diverges and is increasing, then there exists $n \in \mathbb{N}$ such that $a_n > 100$.
4. If $\lim_{n\to\infty} a_n = L$, then $a_n < L + 1$ for all n.
5. If a sequence is non-decreasing and non-increasing, then it is convergent.
6. If a sequence isn't decreasing and isn't increasing, then it is convergent.

Composition law

Write a proof for the following Theorem

Theorem

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Let $L \in \mathbb{R}$.

- IF $\begin{cases} \{a_n\}_{n=0}^{\infty} \longrightarrow L \\ f \text{ is continuous at } L \end{cases}$
- THEN $\{f(a_n)\}_{n=0}^{\infty} \longrightarrow f(L)$.

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1. Write the definition of your hypotheses and your conclusion.
2. Using the definition of your conclusion, figure out the structure of the proof.
3. Do some rough work if necessary.
4. Write a formal proof.

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Critique your proof!

1. Does your proof have the correct structure?
2. Did you introduce all your variables in the right order?
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3. Did you explain what you are doing?
4. Can the proof be fully understood without looking at your rough work?

Squeeze Theorem

1. Write a version of the Squeeze Theorem for convergent sequences.

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2. Write a version of the Squeeze Theorem for sequences divergent to ∞.

Almost increasing

A sequence $\{a_n\}_{n=0}^{\infty}$ is called *2-increasing* when

$$\forall n \in \mathbb{N}, \quad a_n < a_{n+2}.$$

Construct a sequence that is 2-increasing but not increasing.

Proof from the definition of limit

Prove, directly from the definition of limit, that

$$\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1.$$