- Today: Properties of sequences.

- Homework before Wednesday's class: watch videos 11.5, 11.6.

Review: the limit of a sequence

Recall: $\lim_{n\to\infty} a_n = L$ means that

$$\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$$

Find the limit and prove it directly from the definition for

$$a_n = \frac{n}{3n+5}$$

True or False – Monotonic sequences vs functions

Let f be a function with domain $[0, \infty)$.
We define a sequence as $a_n = f(n)$.

1. IF f is increasing, THEN $\{a_n\}_{n=0}^{\infty}$ is increasing.

2. IF $\{a_n\}_{n=0}^{\infty}$ is increasing, THEN f is increasing.

If you think any of them is false,
prove it with a counterexample.

Examples

Construct 8 examples of sequences.
If any of them is impossible, cite a theorem to justify it.

		convergent	divergent
monotonic	bounded	???	???
	unbounded	???	???
not monotonic	bounded	???	???
	unbounded	???	???

A suspicious calculation – What is wrong?

The sequence $\{a_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} & a_0 = 1 \\ \forall n \in N, & a_{n+1} = 1 - a_n \end{cases}$$

has limit $1/2$.

Proof.

- Let $L = \lim_{n \to \infty} a_n$.
- $a_{n+1} = 1 - a_n$
- $\lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} [1 - a_n]$
- $L = 1 - L$
- $L = 1/2$.

A sequence defined by recurrence

Consider the sequence $\{R_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} & R_0 = 1 \\ \forall n \in N, & R_{n+1} = \dfrac{R_n + 2}{R_n + 3} \end{cases}$$

Compute $R_1,\ R_2,\ R_3$.

Is this proof correct?

Let $\{R_n\}_{n=0}^{\infty}$ be the sequence in the previous slide.

Claim:

$\{R_n\}_{n=0}^{\infty} \longrightarrow -1+\sqrt{3}.$

Proof.

- Let $L = \lim_{n\to\infty} R_n$.
- $R_{n+1} = \dfrac{R_n+2}{R_n+3}$
- $\lim_{n\to\infty} R_{n+1} = \lim_{n\to\infty} \dfrac{R_n+2}{R_n+3}$
- $L = \dfrac{L+2}{L+3}$
- $L(L+3) = L+2$
- $L^2+2L-2=0$
- $L = -1 \pm \sqrt{3}$
- L must be positive, so $L = -1+\sqrt{3}$

□

A sequence defined by recurrence

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$$\begin{cases} & R_0 = 1 \\ \forall n \in N, & R_{n+1} = \dfrac{R_n + 2}{R_n + 3} \end{cases}$$

1. Prove $\{R_n\}_{n=0}^{\infty}$ is bounded below by 0.
2. Prove $\{R_n\}_{n=0}^{\infty}$ is decreasing (use induction)
3. Prove $\{R_n\}_{n=0}^{\infty}$ is convergent (use a theorem)
4. Now the calculation in the previous slide is correct, and we can get the value of the limit.