MAT137

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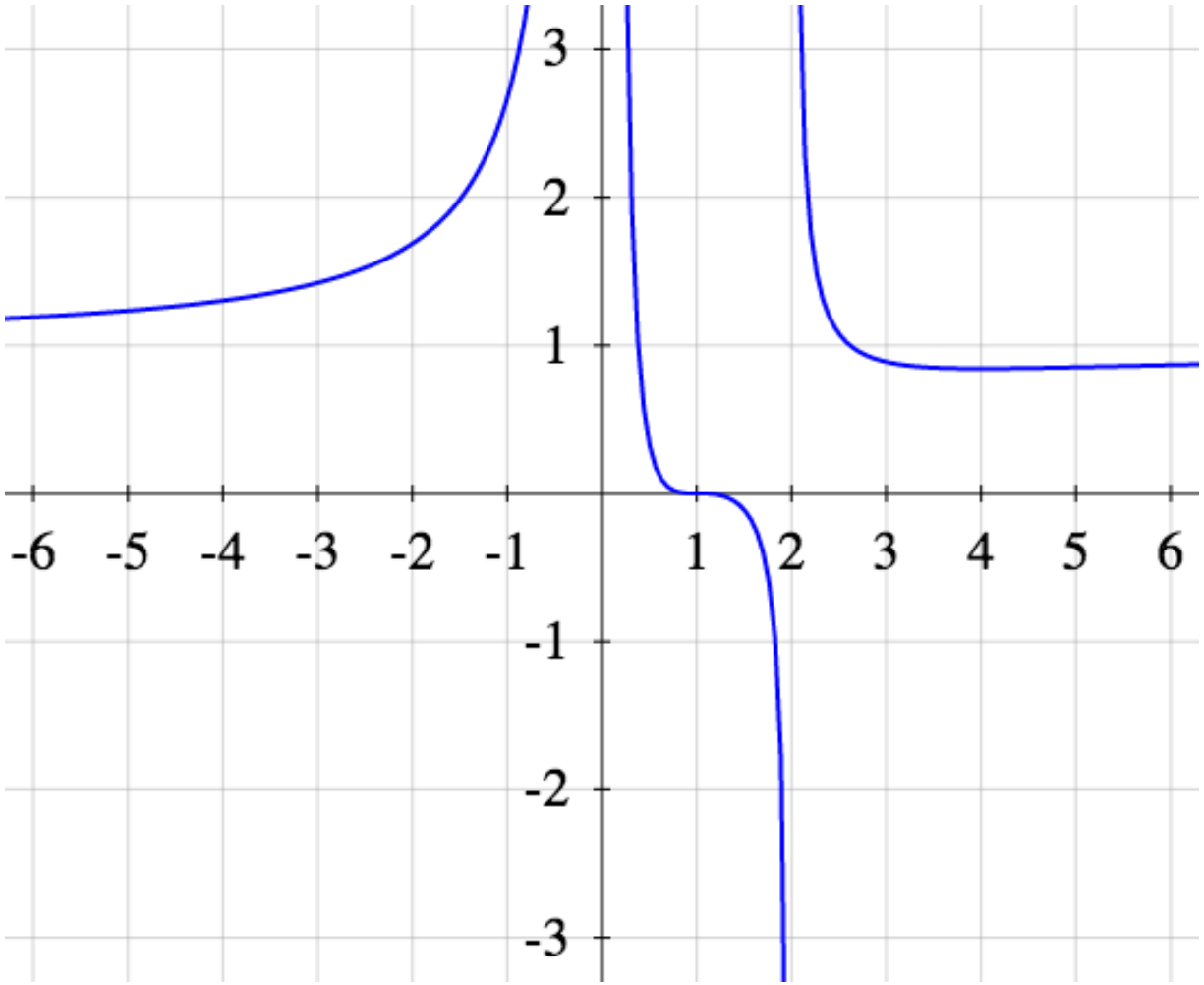
- BUT: review and solve problems on the slides of the last three times (Dec. 1, 6 and 7)
- Watch videos 7.1, 7.2 before next term.

Reminder: Backwards graphing

This is the graph of $y = R(x)$. R is a rational function (a quotient of polynomials). Find its equation.



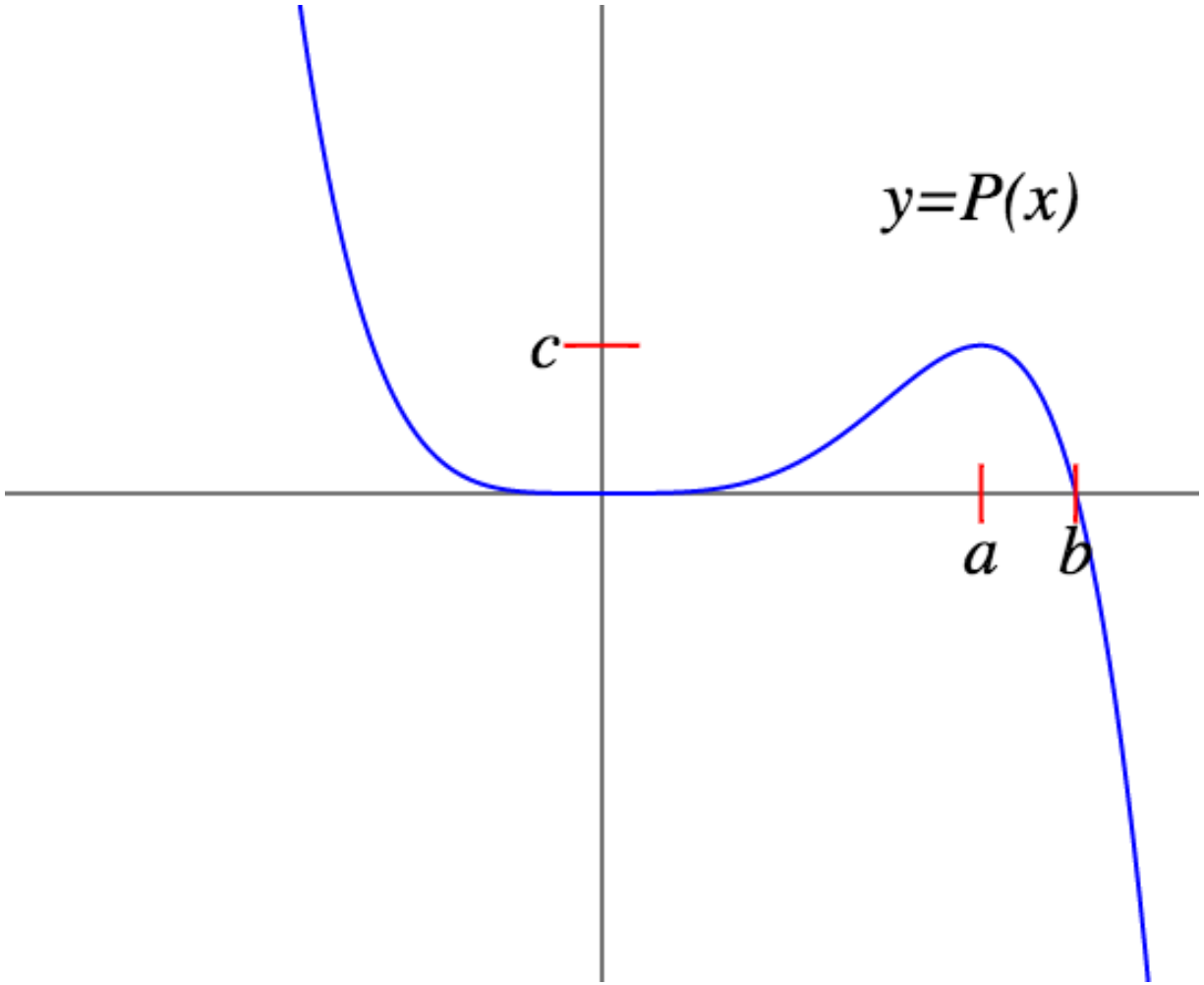
A polynomial from 3 points

Construct a polynomial that satisfies the following three properties at once:

1. It has an inflection point at $x = 2$
2. It has a local extremum at $x = 1$
3. It has y-intercept at $y = 1$.

Backwards graphing 2

Below is the graph of a polynomial P. Notice that it is not at scale. The coordinates in the graph are $a = 24$, $b = 25$, and $c = 1$. Find the equation of P.



A very hard function to graph

The function $G(x) = xe^{1/x}$ is deceiving. To help you out:

$$G'(x) = \frac{x-1}{x}e^{1/x}, \qquad G''(x) = \frac{e^{1/x}}{x^3}$$

1. Carefully study the behaviour as $x \to \pm\infty$.
   You should find an asymptote, but it is not easy.
2. Carefully study the behaviour as $x \to 0^+$ and $x \to 0^-$.
   The two are very different.
3. Use G' to study monotonocity.
4. Use G'' to study concavity.
5. Sketch the graph of G.

Periodic?

Construct a function H that has all the following properties at once:

1. The domain of H is $\mathbb{R}$
2. H is strictly increasing on $\mathbb{R}$
3. H is differentiable on $\mathbb{R}$
4. H' is periodic with period with period 2π
5. H' is not constant

Hint: A function depending on a parameter

Let $a > 0$. Consider the function

$$H(x) = x + a \sin x.$$

Study the monotonicity and concavity of this function, and use it to sketch its graph.
The answer will depend on a. You will obtain two different "types" of graphs, depending on the value of a.

Suggestion: If you have trouble with the function, first restrict it to the domain $0 \leq x \leq 2\pi$, and later expand it to all of $\mathbb{R}$.