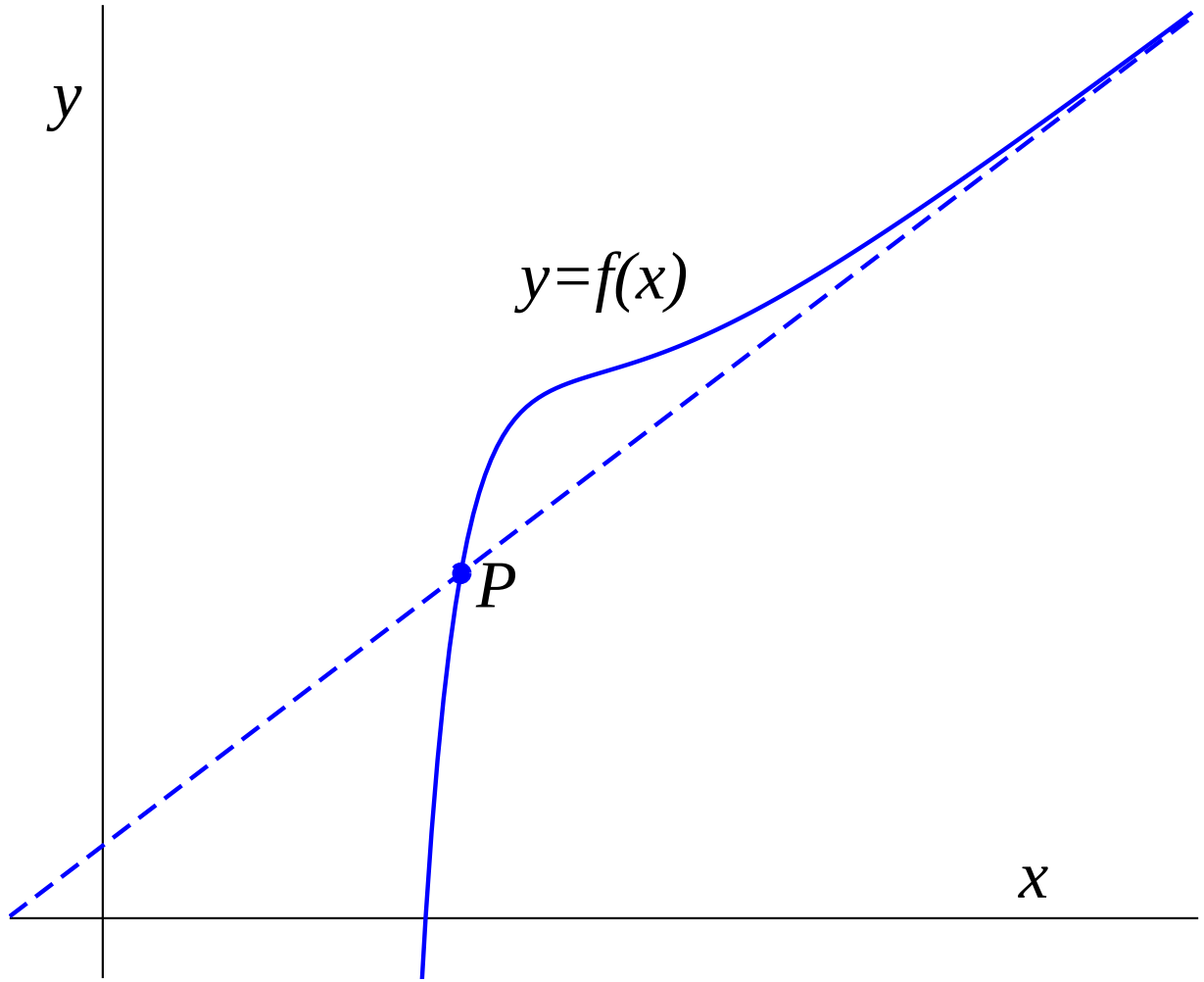
MAT137

- Today: Asymptotes.

- Tomorrow: review videos 6.13 – 6.18.

Reminder: Find the coordinates of P

$$f(x) = 3x + 4 + \frac{2x - 10}{x^2}$$



An equation from the asymptotes

Construct a function f that satisfies all the following conditions at the same time.

- f is a rational function (this means it is a quotient of polynomials).
- The line $y = 1$ is an asymptote of the graph of f.
- The line $x = -1$ is an asymptote of the graph of f.

Hyperbolic tangent

The function tanh, defined by

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

is called the "hyperbolic tangent".

1. Find its two asymptotes
2. Study its monotonicity
3. Study its concavity
4. With this information, sketch its graph.

Unexpected asymptotes

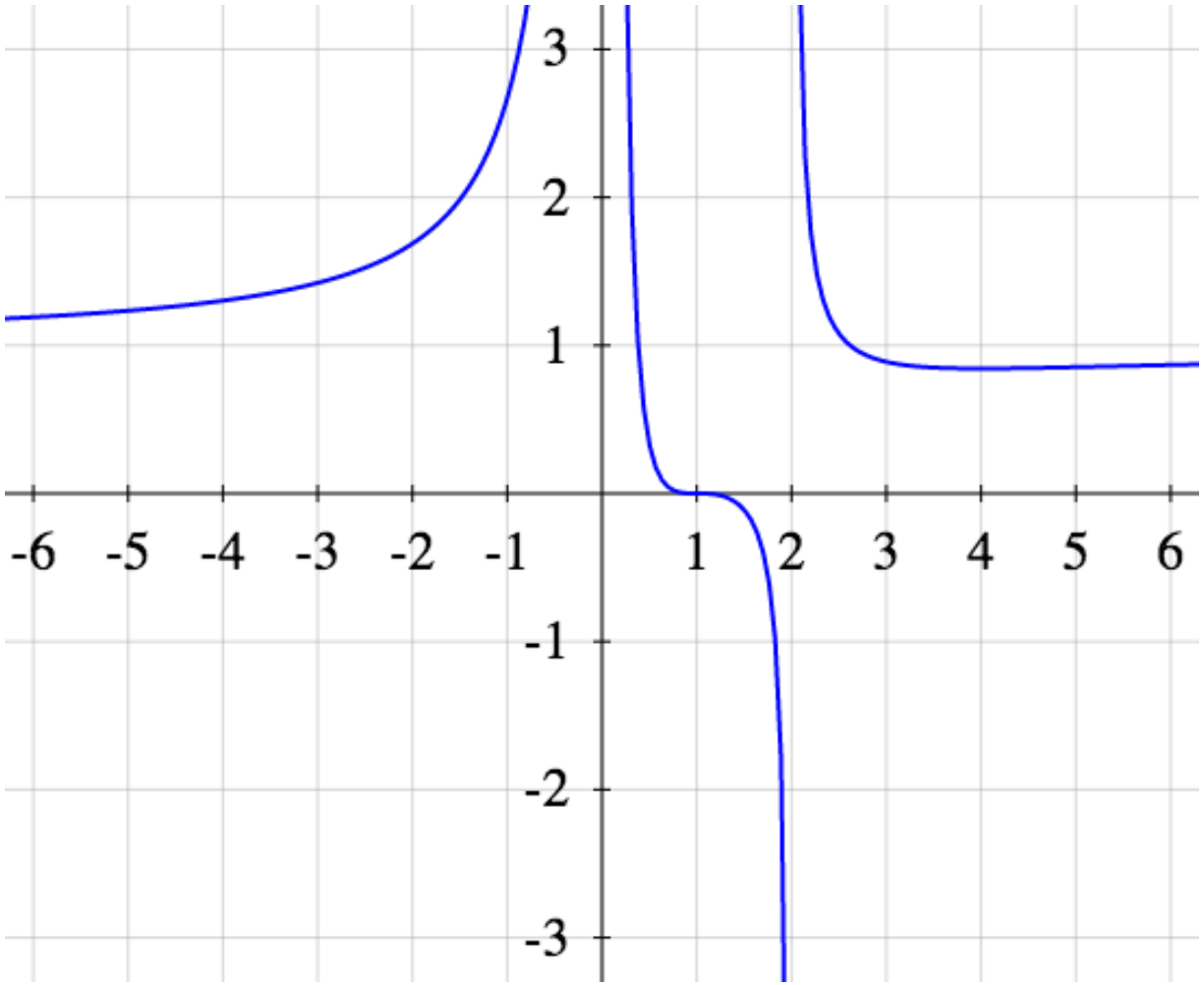
Find the two asymptotes of the function

$$F(x) = x + \sqrt{x^2 + x}$$

Hint: The behaviour as $x \to \infty$ is very different from $x \to -\infty$.

Backwards graphing

This is the graph of $y = R(x)$. R is a rational function (a quotient of polynomials). Find its equation.



A function with fractional exponents

Let $h(x) = \dfrac{x^{2/3}}{(x-1)^{2/3}}$. Its first two derviatives are

$$h'(x) = \frac{-2}{3x^{1/3}(x-1)^{5/3}} \qquad h''(x) = \frac{2(6x-1)}{9x^{4/3}(x-1)^{8/3}}$$

1. Find all asymptotes of h
2. Study the monotonicity of h and local extrema
3. Study the concavity of h and inflection points
4. With this information, sketch the graph of h