- Today: Even more applications and Outroduction.

Add these series

1. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{n+1}{(2n)!}2^n$

2. Find $f(x)=\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!(n+1)}$ What is $f^{(56)}(0)$?

3. $\displaystyle\sum_{n=2}^{\infty}\frac{n(n-1)}{3^n}$

Recall: Challenge

We want to calculate the value of

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\,3^n}$$

Hints:

1. Compute $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$
2. Compute $\dfrac{d}{dx}[\arctan x]$
3. Pretend you can take derivatives and antiderivatives of series the way you can take them of sums. Which series adds up to $\arctan x$?
4. Now attempt the original problem.

Farewell Challenge: division of the stakes

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Note: This problem was the origin of Pascal's triangle. The answer: if one player needs r points to win and the other needs s points to win, the correct division of the stakes is in the ratio: (sum of the first s entries):(sum of the last r entries) in the line of length $r + s$ in Pascal's triangle.

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"Doomsday" = The Day Of The Week for March 0
= Feb.28 or Feb.29 (for leap years)

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Also Doomsdays are 5/9, 9/5, 7/11, 11/7.

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Pi Day 3/14 is also a Doomsday.
Finally, Tuesdays are Feb.0 (=Jan.31) and Jan.3 in 2023.
(In leap years, Doomsdays are Feb.29, Feb.1 (=Jan.32), and Jan.4.)

Watch this video

What to read next?

1. Many remarkable stories on Newton and Leibniz:

Vladimir Arnold: *Huygens and Barrow, Newton and Hooke: pioneers in mathematical analysis and catastrophe theory from evolvents to quasicrystals* 1990.

2. Many challenging puzzles and problems (many of them are very hard!):

Vladimir Arnold: *Problems for children from 5 to 15*

Problems for kids from 5 to 15