MAT137

- Today: More applications.

- No homework before Thursday's class!

Limits

Compute these limits by writing out the first few terms of the Maclaurin series of numerator and denominator:

1. $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

2. $\lim_{x\to 0} \dfrac{6\sin x - 6x + x^3}{x^5}$

3. $\lim_{x\to 0} \dfrac{x^2 \sin x^2 - x^4}{e^{x^8} - 1}$

4. Find a value of $a \in \mathbb{R}$ such that the limit

$$\lim_{x\to 0} \frac{e^{\sin x} - e^x + ax^3}{x^4}$$

exists and is not 0. Then compute the limit.

Estimation

I want to estimate these two numbers

$$A = \sin 1, \qquad B = \ln 0.9$$

with a small error (e.g. smaller than 0.001).

1. Use Taylor series to write A and B as infinite sums.
2. If you want to estimate A or B with a partial sum, the fastest way is to use different theorems for A and B. What are they?
3. Estimate them.

Come to the dark side

Boris Khesin is preparing a tricky question for the MAT137 final exam. He wants to ask students to compute a limit of the form

$$\lim_{x \to 0} \frac{Ae^{x^4} + Be^{-x^4} + C\cos(x^4) + x^n}{x^m},$$

where A, B, and C will be specific real numbers, and m and n will be specific positive integers. He wants to choose these five constants so that the limit will be very difficult for those who try to solve it using L'Hôpital's Rule (specifically, he wants the calculation to require as many uses of L'Hôpital's Rule as possible) but easier to do using Taylor series. In addition, he does not want to answer to be 0 or ∞ or $-\infty$ or "DNE", because those are all answers that students could guess by accident.

Calculate the value of A, B, C, n, m, and the limit.