Instructor: B. Khesin

Graduate course MAT 1121HF
"Lie Groups and Hamiltonian Dynamical Systems"
Syllabus:

I. Main notions: Lie groups, Lie algebras, adjoint and coadjoint representations.

1. Lie groups and Lie algebras.
2. The adjoint representation.
3. Group adjoint orbits.
4. The coadjoint representation and orbits.
5. Central extensions.
6. The Lie–Poisson (or Euler) equations for Lie groups.
7. Bihamiltonian systems.

II. Geometry of infinite-dimensional Lie groups and their orbits.

1. Affine Kac–Moody Lie algebras and groups.
   1.1. Definition of the affine Kac–Moody Lie algebras.
   1.2. Affine Lie groups.
   1.3. Their coadjoint orbits.
   1.4. The quotient (WZW) construction of the affine groups.
2. The Virasoro algebra and group.
   2.1. Definitions.
   2.2. Diffeomorphisms of the circle.
   2.3. The Virasoro coadjoint action.
   2.4. Virasoro coadjoint orbits.
   2.5. The Virasoro group and Korteweg-de Vries equation.
   2.6. Bihamiltonian structure of the KdV.
3. Groups of diffeomorphisms.
   3.1. The Lie group of volume-preserving diffeomorphisms and its Lie algebra.
   3.2. Coadjoint invariants.
   3.3. Other diffeomorphism groups.
4. The double loop (or elliptic) Lie groups and Lie algebras.
   4.1. Definitions.
   4.2. Classification of coadjoint orbits.
   4.3. Monodromy and holomorphic loop algebras.

7. Groups of (pseudo)differential operators.
   7.1. Pseudodifferential operators and cocycles on them.
   7.2. The Lie group of pseudodifferential operators of complex degree.
   7.3. Integrable KP-KdV hierarchies.

III. Poisson structures on moduli spaces.

1. Definition and integrability of holomorphic bundles.
2. Moduli spaces of flat connections.
3. The Poincaré residue and Cauchy–Stokes formula.
4. Moduli spaces of holomorphic bundles.

IV. Around the Chern–Simons functional.

1. A reminder on the Lagrangian formalism.
2. Main example: the Chern–Simons action functional.
3. The holomorphic Chern–Simons functional.
4. The Chern–Simons functional and linking numbers.

References:

1. B. Khesin and R. Wendt "The geometry of infinite-dimensional groups," Springer (2008), to appear, see
http://www.math.toronto.edu/khesin/papers/Khesin_Proofs.pdf

2. A. Pressley and G. Segal: "Loop Groups," Clarendon Press, Oxford (1986)