Instructor: B. Khesin

**Graduate course MAT 1126H**
**2026–2027**
**"Geometric Fluid Dynamics"**

Overview:

This course deals with various problems in geometry, Lie theory, and Hamiltonian systems, motivated by hydrodynamics and magnetohydrodynamics. We discuss the dynamics of an ideal fluid from the group-theoretic and Hamiltonian points of view. We cover geometry of conservation laws of the Euler equation, point vortex approximations, topology of steady flows and their nonlinear stability, relation of the energy and helicity of vector fields, geometry of diffeomorphism groups, relation to vortex sheets, as well descriptions of magnetohydrodynamics and of the Korteweg-de Vries equation in the Lie group framework.

**Syllabus:**

I. Main notions in Lie groups and Lie algebras.

II. The Euler equation of an ideal fluid as a geodesic flow.
General Euler-Arnold equations.

III. The Hamiltonian framework for the Euler equations:
Equations on the dual Lie algebra, Poisson structures.

IV. Examples:
Ideal hydrodynamics, Magnetohydrodynamics (MHD), the Korteweg-deVries and Camassa-Holm equations (on Virasoro groups), $\beta$-plane equation in meteorology, Landau-Lifschits equation.

V. Conservation laws for fluids:
First integrals for ideal and compressible fluids. Point vortex approximations in 2D. Vortex filament equation and the Marsden-Weinstein structures on the spaces of curves.

VI. Steady solutions in 2D and 3D.
Bernoulli function, related variational problems, restrictions on topology of steady solutions, Arnold's stability criterion.

VII. Topology bounds the energy of a field.
An ergodic interpretation of helicity (asymptotic Hopf invariant), energy estimates, Sakharov–Zeldovich problem.

VIII. Other applications (time permitted):
Differential geometry of diffeomorphism groups: Otto calculus and optimal transport, geometry of vortex sheets and shock waves.

**References:**

V.Arnold and B.Khesin: *Topological Methods in Hydrodynamics,* Appl. Math. Series, v. 125, Springer-Verlag, 1998, Second extended edition: 2021.

J.Marsden and T.Ratiu: *Introduction to Mechanics and Symmetry,* Texts in Applied Math., v. 17, Springer-Verlag, 1994/1999.

**Prerequisites:**

MAT367 (Differential geometry)
MAT351/APM346/APM446/ (Partial Differential Equations)
MAT301/MAT347 (Groups and symmetries)