How to Talk Mathematics

Boris Khesin

(Organizational meeting of the seminar)

UofT Math

Our goals

Our plan is to cover 3 directions:

- **teaching (math) classes**

- **giving good math talks**

- **good math writing**

with the emphasis on the first one.

To be run biweekly (sometimes weekly), invite our profs to share their know-hows about teaching.

Time: Tuesday 4pm, sometimes (next week!) Monday 4pm

Organizers: Boris Khesin, Grigorii Taroian, Tara Stojimirovic

Several tips on lecturing

1) How to make a large audience silent?

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

2) Invite students to ask questions, to interrupt,

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

2) Invite students to ask questions, to interrupt,

but keep control if it is too frequent or too distracting.

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

2) Invite students to ask questions, to interrupt,

but keep control if it is too frequent or too distracting.

3) Maybe give puzzles during the last 5 min on Fridays (Martin Gardner's style)

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

2) Invite students to ask questions, to interrupt,

but keep control if it is too frequent or too distracting.

3) Maybe give puzzles during the last 5 min on Fridays (Martin Gardner's style)
(Martin Gardner was best known for his "Mathematical Games" column in *Scientific American*, which ran for 25 years)

Several tips on lecturing

1) How to make a large audience silent?

My tool: Sh-sh-sh-...!

2) Invite students to ask questions, to interrupt,

but keep control if it is too frequent or too distracting.

3) Maybe give puzzles during the last 5 min on Fridays (Martin Gardner's style)
(Martin Gardner was best known for his "Mathematical Games" column in *Scientific American*, which ran for 25 years)

4) Mild jokes?

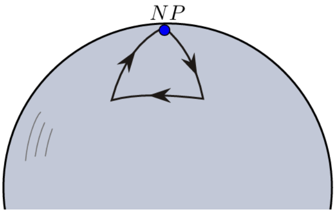
Puzzles on Fridays: examples

Puzzle 1: A hunter started from his tent and went 10 km strictly south, then 10 km strictly west, then 10 km strictly north, and arrived at his original tent. Where was his tent?

Puzzles on Fridays: examples

Puzzle 1: A hunter started from his tent and went 10 km strictly south, then 10 km strictly west, then 10 km strictly north, and arrived at his original tent. Where was his tent?

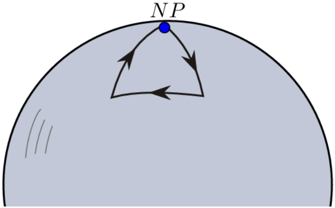
One evident solution is the North Pole NP. But is it unique?



Puzzles on Fridays: examples

Puzzle 1: A hunter started from his tent and went 10 km strictly south, then 10 km strictly west, then 10 km strictly north, and arrived at his original tent. Where was his tent?

One evident solution is the North Pole NP. But is it unique?



Puzzle 2: A hunter started from his tent and went 10 km south, then 10 km west, then 10 km north, then 10 km east, and arrived at his original tent. Where was his tent?

What is it?

"How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics..." (c) Sir James Hopwood Jeans

What is it?

“How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics...” (c) Sir James Hopwood Jeans

This is a mnemonic for $\pi = 3.14159265358979...$

What is it?

“How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics...” (c) Sir James Hopwood Jeans

This is a mnemonic for $\pi = 3.14159265358979...$

“How I need a drink, alcoholic of course, after the heavy lectures involving vicious Calculus!” (c) Boris Khesin

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.
3) Think through the talk timing.

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.

3) Think through the talk timing.

4) Alexandre Kirillov used to say: “If you have a good result, it can be formulated in 3 min. If it is a bit worse, then in 5 min, etc.”

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.
3) Think through the talk timing.

4) Alexandre Kirillov used to say: “If you have a good result, it can be formulated in 3 min. If it is a bit worse, then in 5 min, etc.”

5) Many more suggestions from Halmos “How to talk Mathematics”.

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.

3) Think through the talk timing.

4) Alexandre Kirillov used to say: "If you have a good result, it can be formulated in 3 min. If it is a bit worse, then in 5 min, etc."

5) Many more suggestions from Halmos "How to talk Mathematics". For instance, "warning: if the dry run over your talk notes takes an hour, then the actual delivery will take an hour an a half."

Tips on giving a talk

Of course, it is a matter of personal taste. My suggestions:

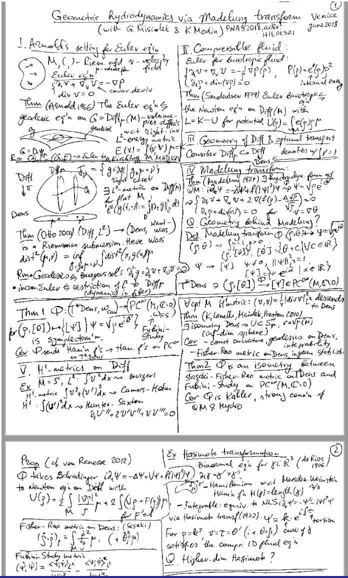
1) Never ask for questions !

2) The talk preparation - think of what to write on a blackboard, not what to say.
3) Think through the talk timing.

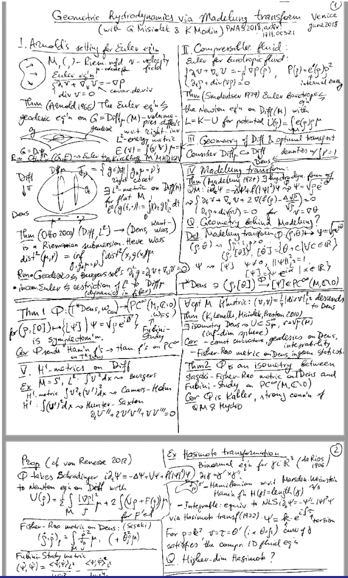
4) Alexandre Kirillov used to say: "If you have a good result, it can be formulated in 3 min. If it is a bit worse, then in 5 min, etc."

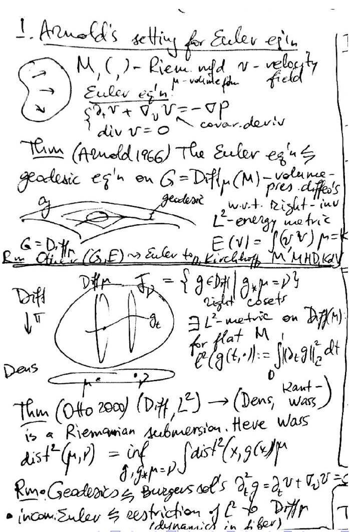
5) Many more suggestions from Halmos "How to talk Mathematics". For instance, "warning: if the dry run over your talk notes takes an hour, then the actual delivery will take an hour an a half."
"Make a lecture flexible, use portable modules. My notes of a lecture usually consists of about 20 paragraphs (2-4 minutes each), and the last half the paragraphs are omitable. I sail through the first half of the period with no worries: I am sure that everything that I must say will get said."

My preparation for a blackboard talk



My preparation for a blackboard talk





Random thoughts on giving a talk

1) Even useless figures can be useful.

Random thoughts on giving a talk

1) Even useless figures can be useful.

2) Keep examples handy and keep reminding of them in the talk. The listeners will keep understanding only as long as they have at least one example to test the theory, and they will not be able to move beyond the point in the talk where their last example disappears, due to some new assumptions.

Random thoughts on giving a talk

1) Even useless figures can be useful.

2) Keep examples handy and keep reminding of them in the talk. The listeners will keep understanding only as long as they have at least one example to test the theory, and they will not be able to move beyond the point in the talk where their last example disappears, due to some new assumptions.

3) Use the Stanislavsky system: think of what you would like to see in the talk as a listener. For instance, try to give shorter answers to the audience questions.

Random thoughts on giving a talk

1) Even useless figures can be useful.

2) Keep examples handy and keep reminding of them in the talk. The listeners will keep understanding only as long as they have at least one example to test the theory, and they will not be able to move beyond the point in the talk where their last example disappears, due to some new assumptions.

3) Use the Stanislavsky system: think of what you would like to see in the talk as a listener. For instance, try to give shorter answers to the audience questions.

4) Remember: thinking while standing at the blackboard is $e \approx 2.7$ times slower than while sitting!

Random thoughts on giving a talk

1) Even useless figures can be useful.

2) Keep examples handy and keep reminding of them in the talk. The listeners will keep understanding only as long as they have at least one example to test the theory, and they will not be able to move beyond the point in the talk where their last example disappears, due to some new assumptions.

3) Use the Stanislavsky system: think of what you would like to see in the talk as a listener. For instance, try to give shorter answers to the audience questions.

4) Remember: thinking while standing at the blackboard is $e \approx 2.7$ times slower than while sitting!

5) A tip for chairing a protracted talk: if the speaker does not stop while his time is long over, make a step toward him. In a minute make another step. And another...

Several tips on writing a paper

1) Read "How to write mathematics" by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

Several tips on writing a paper

1) Read “How to write mathematics” by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

2) Read “A Mathematician’s Miscellany” by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

Several tips on writing a paper

1) Read "How to write mathematics" by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

2) Read "A Mathematician's Miscellany" by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

3) From V.Arnold: Titles and subtitles have to be meaningful to the extent that the table of contents becomes a "mini-paper" on its own. The same holds for figures: the captions have to carry messages without relying on the main text.

Several tips on writing a paper

1) Read “How to write mathematics” by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

2) Read “A Mathematician’s Miscellany” by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

3) From V.Arnold: Titles and subtitles have to be meaningful to the extent that the table of contents becomes a “mini-paper” on its own. The same holds for figures: the captions have to carry messages without relying on the main text.

4) Introduction is a minipaper! Try formulating your main result there, maybe skipping details. Many readers will not go further.

Several tips on writing a paper

1) Read "How to write mathematics" by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

2) Read "A Mathematician's Miscellany" by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

3) From V.Arnold: Titles and subtitles have to be meaningful to the extent that the table of contents becomes a "mini-paper" on its own. The same holds for figures: the captions have to carry messages without relying on the main text.

4) Introduction is a minipaper! Try formulating your main result there, maybe skipping details. Many readers will not go further.

5) Again and again: Keep examples handy and keep reminding of them in the text.

Several tips on writing a paper

1) Read "How to write mathematics" by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

2) Read "A Mathematician's Miscellany" by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

3) From V.Arnold: Titles and subtitles have to be meaningful to the extent that the table of contents becomes a "mini-paper" on its own. The same holds for figures: the captions have to carry messages without relying on the main text.

4) Introduction is a minipaper! Try formulating your main result there, maybe skipping details. Many readers will not go further.

5) Again and again: Keep examples handy and keep reminding of them in the text.

6) Always name the notion next to its notation: point P, variety X (to recall your notation to the reader).

From Arnold: captions = minipaper

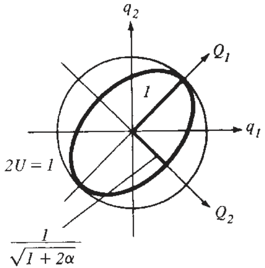


Figure 80 Configuration space of the connected pendulums

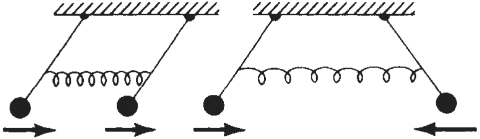


Figure 81 Characteristic oscillations of the connected pendulums

Table of Contents of Halmos' "How to write Math"

0. Preface
1. There is no recipe and what it is
2. Say something
3. Speak to someone
4. Organize first
5. Think about the alphabet
6. Write in spirals
7. Organize always
8. Write good English
9. Honesty is the best policy
10. Down with the irrelevant and trivial
11. Do and do not repeat

$< \ldots >$

19. Stop
20. The last word

From Littlewood: 2 proofs of the Weierstrass Theorem I

* A famous theorem of Weierstrass says that a function $f(x_1, x_2)$ continuous in a rectangle R, can be uniformly approximated to by a sequence of polynomials in x_1, x_2. It is valid in n dimensions, and the beginner will give what follows, but in $x_1, x_2, \ldots, x_n$; $x_1', x_2', \ldots, x_n'$. The proof, an audacious combination of ideas, is in 2 parts; the second cannot be badly mauled and I give it at the end. Here is the beginner's proof of the first part. I am indebted to Dr. Flett for one or two happy misimprovements, and for additional realism have left some incidental misprints uncorrected.

From Littlewood: 2 proofs of the Weierstrass Theorem II

With $f(x_1, x_2)$ continuous in $(-a \leqslant x_1 \leqslant a,\ -b \leqslant x_2 \leqslant b)$, let $c>0$ and define a function $f_1(x_1, x_2)$ by

$$f_1(x_1, x_2) = \begin{cases} f(-a, b)\ (-c-a \leqslant x_1 \leqslant -a,\ b \leqslant x_2 \leqslant b+c) \\ f(x_1, b)\ (-a \leqslant x_1 \leqslant a,\ b \leqslant x_2 \leqslant b+c) \\ f(a, b)\ (-a \leqslant x_1 \leqslant a+c,\ b \leqslant x_2 \leqslant b+c) \\ f(-a, x_2)\ (-a-c \leqslant x_1 \leqslant -a,\ -b \leqslant x_2 \leqslant b) \\ f(x_1, x_2)\ (-a \leqslant x_1 \leqslant a,\ -b \leqslant x_2 \leqslant b) \\ f(a, x_2)\ (a \leqslant x_1 \leqslant a+c,\ -b \leqslant x_2 \leqslant b) \\ f(a, -b)\ (-a-c \leqslant x_1 \leqslant -a,\ -b-c \leqslant x_2 \leqslant -b) \\ f(x_1, -b)\ (-a \leqslant x_1 \leqslant a,\ -b-c \leqslant x_2 \leqslant -b) \\ f(-a, -b)\ (-a-c \leqslant x_1 \leqslant -a,\ -b-c \leqslant x_2 \leqslant -b). \end{cases}$$

It can easily be shown that $f_1(x_1, x_2)$ is continuous in

$$(-a-c \leqslant x_1 \leqslant a+c,\ -b-c \leqslant x_2 \leqslant b+c).$$

For points (x_1, x_2) of R define

$$\phi_n(x_1, x_2) = \pi^{-1} n \int_{-a-c}^{a+c} dx'_1 \int_{-b-c}^{b+c} f_1(x'_1, y'_1) \exp[-n\{(x'_1 - x_1)^2 + (y'_1 - y_1)^2\}] dx'_2.$$

We shall show [this is the first half referred to above] that

(1) $\phi_n(x_1, x_2) \to f(x_1, x_2)$ as $n \to \infty$, uniformly for (x_1, x_2) of R.

There is a $\delta(\epsilon)$ such that $|f_1(x_1'', x_2'') - f_1(x_1', x_2')| < \epsilon$ provided that (x_1', x_2') and (x_1'', x_2'') belong to $(-a-c \leqslant x_1' \leqslant a+c,\ -b-c \leqslant x_2' \leqslant b+c)$ and satisfy $|x_1'' - x_1'| < \delta(\epsilon)$ and $|x_2'' - x_2'| < \delta(\epsilon)$. Let

$$n_0 = n_0(\epsilon) = \text{Max}\,([c^3]+1,\ [\delta^{-3}(\epsilon)]+1),$$

and let $n > n_0$. Then $-a-c < x_1 - n^{-\frac{1}{3}} < x_1 + n^{-\frac{1}{3}} < a+c$, $-b-c < x_2 - n^{-\frac{1}{3}} < x_2 + n^{-\frac{1}{3}} < b+c$, and we have

$$\phi_n(x_1, x_2) = \pi^{-1} n \Bigg[\int_{-a+c}^{a+c} dx_1' \int_{x_2+n^{-\frac{1}{3}}}^{b+c} dx_2' + \int_{-a-c}^{a+c} dx_1' \int_{x_2-n^{-\frac{1}{3}}}^{x_2+n^{-\frac{1}{3}}} dx_2' + \int_{x_1-n^{-\frac{1}{3}}}^{x_1+n^{-\frac{1}{3}}} dx_1' \int_{x_2-n^{-\frac{1}{3}}}^{x_2+n^{-\frac{1}{3}}} dx_2'$$

From Littlewood: 2 proofs of the Weierstrass Theorem II

With $f(x_1, x_2)$ continuous in $(-a\leqslant x_1\leqslant a,\ -b\leqslant x_2\leqslant b)$, let $c>0$ and define a function $f_1(x_1, x_2)$ by

$$f_1(x_1,x_2)=\begin{cases} f(-a,b)\ (-c-a\leqslant x_1\leqslant -a,\ b\leqslant x_2\leqslant b+c)\\ f(x_1,b)\ (-a\leqslant x_1\leqslant a,\ b\leqslant x_2\leqslant b+c)\\ f(a,b)\ (-a\leqslant x_1\leqslant a+c,\ b\leqslant x_2\leqslant b+c)\\ f(-a,x_2)\ (-a-c\leqslant x_1\leqslant -a,\ -b\leqslant x_2\leqslant b)\\ f(x_1,x_2)\ (-a\leqslant x_1\leqslant a,\ -b\leqslant x_2\leqslant b)\\ f(a,x_2)\ (a\leqslant x_1\leqslant a+c,\ -b\leqslant x_2\leqslant b)\\ f(a,-b)\ (-a-c\leqslant x_1\leqslant -a,\ -b-c\leqslant x_2\leqslant -b)\\ f(x_1,-b)\ (-a\leqslant x_1\leqslant a,\ -b-c\leqslant x_2\leqslant -b)\\ f(-a,-b)\ (-a-c\leqslant x_1\leqslant -a,\ -b-c\leqslant x_2\leqslant -b).\end{cases}$$

It can easily be shown that $f_1(x_1, x_2)$ is continuous in

$$(-a-c\leqslant x_1\leqslant a+c,\ -b-c\leqslant x_2\leqslant b+c).$$

For points (x_1, x_2) of R define

$$\phi_n(x_1,x_2)=\pi^{-1}n\int_{-a-c}^{a+c}dx'_1\int_{-b-c}^{b+c}f_1(x'_1,y'_1)\exp[-n\{(x'_1-x_1)^2+(y'_1-y_1)^2\}]dx'_2.$$

We shall show [this is the first half referred to above] that

(1) $\phi_n(x_1, x_2)\to f(x_1, x_2)$ as $n\to\infty$, uniformly for (x_1, x_2) of R.

There is a $\delta(\epsilon)$ such that $|f_1(x_1'', x_2'')-f_1(x_1', x_2')|<\epsilon$ provided that (x_1', x_2') and (x_1'', x_2'') belong to $(-a-c\leqslant x_1'\leqslant a+c,\ -b-c\leqslant x_2'\leqslant b+c)$ and satisfy $|x_1''-x_1'|<\delta(\epsilon)$ and $|x_2''-x_2'|<\delta(\epsilon)$. Let

$$n_0=n_0(\epsilon)=\text{Max}\ ([c^3]+1,\ [\delta^{-3}(\epsilon)]+1),$$

and let $n>n_0$. Then $-a-c<x_1-n^{-\frac{1}{3}}<x_1+n^{-\frac{1}{3}}<a+c$, $-b-c<x_2-n^{-\frac{1}{3}}<x_2+n^{-\frac{1}{3}}<b+c$, and we have

$$\phi_n(x_1,x_2)=\pi^{-1}n\Bigg[\int_{-a+c}^{a+c}dx_1'\int_{x_2+n^{-\frac{1}{3}}}^{b+c}dx_2'+\int_{-a-c}^{a+c}dx_1'\int_{x_2-n^{-\frac{1}{3}}}^{x_2+n^{-\frac{1}{3}}}dx_2'+\int_{x_1-n^{-\frac{1}{3}}}^{x_1+n^{-\frac{1}{3}}}dx_1'\int_{x_2-n^{-\frac{1}{3}}}^{x_2+n^{-\frac{1}{3}}}dx_2'$$

(2) $=T_1+T_2+\cdots+T_5$,

say. In T_1 we have $|f_1(x_1', x_2')|<K$, $\exp[\ \ ]\leqslant\exp(-n.n^{-\frac{2}{3}})$, and so

(3) $|T_1|<\epsilon(n>n_1(\epsilon))$:

Similarly [1]

(4) $|T_2|,\ |T_4|,\ |T_5|<\epsilon\quad(n>n_2(\epsilon))$.

In T_3 write $x_1'=x_1+x_1''$, $x_2'=x_2+x_2''$. Since $|x_1''|\leqslant n^{-\frac{1}{3}}$, $|x_2''|\leqslant n^{-\frac{1}{3}}$ in the range concerned we have

(5) $|f_1(x_1+x_1'', x_2+x_2'')-f_1(x_1, x_2)|<\epsilon\ (n>\delta^{-3}(\epsilon))$.

Now in T_3 we have $f_1(x_1, x_2)=f(x_1, x_2)$. Hence

(6) $T_3=T_{3,1}+T_{3,2}$, where

(7) $$T_{3,1}=\pi^{-1}nf(x_1,x_2)\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_1''\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_2''\exp\big[-n(x_1''^2+x_2''^2)\big],$$

(8) $$T_{3,2}=\pi^{-1}n\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_1''\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_2''\epsilon\{f_1(x_1+x_1'', x_2+x_2'')-f_1(x_1,x_2)\}\exp\big[-n(x_1''^2+x_2''^2)\big].$$

We have, for $n>\text{Max}(n_0, n_1, n_2)$,

(9) $$|T_{3,2}|\leqslant\pi^{-1}n\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_1''\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}dx_2''\epsilon\exp\Big[-n\Big(x_1''^2+x_2''^2\Big)\Big]$$
$$\leqslant\pi^{-1}\epsilon n\int_{-\infty}^{\infty}dx_1''\int_{-\infty}^{\infty}dx_2''\exp\Big[-n\Big(x_1''^2+x_2''^2\Big)\Big]=\epsilon.$$

Also the double integral in (7) is

(10) $$\left(\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}}e^{-nu^2}du\right)^2$$

From Littlewood: 2 proofs of the Weierstrass Theorem III

HOWLERS, MISPRINTS, ETC. 33

Now $$\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}} e^{-nu^2}du = 2\int_o^{n^{-\frac{1}{3}}} = 2\int_o^{\infty} - 2\int_{n^{-\frac{1}{3}}}^{\infty}$$
$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}} - 2\int_o^{\infty} e^{-n(n^{-\frac{1}{3}}+t)^2}dt$$
$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}} + O\left(e^{-n^{\frac{1}{3}}}\int_o^{\infty} e^{-2n^{\frac{2}{3}}t}dt\right)$$
$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}}\left(1 + O\left(n^{-\frac{1}{6}}e^{-n^{\frac{1}{3}}}\right)\right).$$

Hence it is easily seen that

$$\left|\left(\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}} e^{-nu^2}du\right)^2 - n^{-1}\pi\right| = \left|n^{-1}\pi\{1 + O(n^{-\frac{1}{6}}e^{-n^{\frac{1}{3}}})\} - n^{-1}\pi\right|$$
$$< \epsilon \ (n > n_3(\epsilon)).$$

Hence from (10) and (7)

(11) $|T_{3,1} - f(x_1, x_2)| < \mathrm{K}\epsilon \ (n > \mathrm{Max}(n_0, n_1, n_2, n_3))$.

From (2) to (11) it follows that

$$|\phi_n(x_1, x_2) - f(x_1, x_2)| < \mathrm{K}\epsilon \ (n > \mathrm{Max}(n_0, n_1, n_2, n_3)),$$

and we have accordingly proved (1).

From Littlewood: 2 proofs of the Weierstrass Theorem III

Now $$\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}} e^{-nu^2}du = 2\int_{o}^{n^{-\frac{1}{3}}} = 2\int_{o}^{\infty} - 2\int_{n^{-\frac{1}{3}}}^{\infty}$$

$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}} - 2\int_{o}^{\infty} e^{-n(n^{-\frac{1}{3}}+t)^2}dt$$

$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}} + O\left(e^{-n^{\frac{1}{3}}}\int_{o}^{\infty} e^{-2n^{\frac{2}{3}}t}dt\right)$$

$$= n^{-\frac{1}{2}}\pi^{\frac{1}{2}}\left(1 + O\left(n^{-\frac{1}{6}}e^{-n^{\frac{1}{3}}}\right)\right).$$

Hence it is easily seen that

$$\left|\left(\int_{-n^{-\frac{1}{3}}}^{n^{-\frac{1}{3}}} e^{-nu^2}du\right)^2 - n^{-1}\pi\right| = \left|n^{-1}\pi\{1 + O(n^{-\frac{1}{6}}e^{-n^{\frac{1}{3}}})\} - n^{-1}\pi\right|$$
$$< \epsilon \ (n > n_3(\epsilon)).$$

Hence from (10) and (7)

(11) $|T_{3,1} - f(x_1, x_2)| < K\epsilon \ (n > \mathrm{Max}(n_0, n_1, n_2, n_3))$.

From (2) to (11) it follows that

$$|\phi_n(x_1, x_2) - f(x_1, x_2)| < K\epsilon \ (n > \mathrm{Max}(n_0, n_1, n_2, n_3)),$$

and we have accordingly proved (1).

[1] I disclose at this point that in T_2 $\int_{-a-c}^{a+c}$ is a 'slip' for $\int_{-a-c}^{x_1 - n^{-\frac{1}{3}}}$. One slip is practically certain in this style of writing, generally devil-inspired.

From Littlewood: 2 proofs of the Weierstrass Theorem IV

A civilized proof is as follows.

Extend the definition of $f(x, y)$ to a larger rectangle R_+; e.g. on AB f is to be $f(A)$, and in the shaded square it is to be $f(C)$. The new f is continuous in R_+. Define, for (x, y) of R,

$$\text{(i)} \qquad \phi_n(x, y) = \iint_{R_+} f(\xi, \eta) \mathrm{E} \, d\xi \, d\eta \Big/ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{E} \, d\xi \, d\eta,$$

where $\mathrm{E} = \exp[-n\{(\xi - x)^2 + (\eta - y)^2\}]$. The denominator is the constant πn^{-1} (independent of x, y); hence (i) is equivalent to

$$\text{(ii)} \qquad \phi_n(x, y) = \pi^{-1} n \iint_{R_+} f(\xi, \eta) \mathrm{E} \, d\xi \, d\eta.$$

The contributions to the numerator and to the denominator in (i) of (ξ, η)'s outside the square $S = S(x, y)$ of side $n^{-\frac{1}{3}}$

c

From Littlewood: 2 proofs of the Weierstrass Theorem IV

A civilized proof is as follows.

Extend the definition of $f(x, y)$ to a larger rectangle R_+; e.g. on AB f is to be $f(A)$, and in the shaded square it is to be $f(C)$. The new f is continuous in R_+. Define, for (x, y) of R,

$$\text{(i)} \qquad \phi_n(x, y) = \iint_{R_+} f(\xi, \eta)\mathrm{E}\,d\xi d\eta \Big/ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\,d\xi d\eta,$$

where $\mathrm{E} = \exp[-n\{(\xi-x)^2+(\eta-y)^2\}]$. The denominator is the constant πn^{-1} (independent of x, y); hence (i) is equivalent to

$$\text{(ii)} \qquad \phi_n(x, y) = \pi^{-1} n \iint_{R_+} f(\xi, \eta)\mathrm{E}\,d\xi d\eta.$$

The contributions to the numerator and to the denominator in (i) of (ξ, η)'s outside the square $S = S(x, y)$ of side $n^{-\frac{1}{3}}$

c

round (x, y) are exponentially small. The denominator itself being πn^{-1} we have (o's uniform)

$$\phi_n(x, y) = \left(\iint_S f(\xi, \eta)\mathrm{E}\,d\xi d\eta \Big/ \iint_S \mathrm{E}\,d\xi d\eta\right) + o(1).$$

S being small, the $f(\xi, \eta)$ in the last numerator is $f(x, y)+o(1)$; so finally the ϕ_n defined by (ii) satisfies $\phi_n(x, y) = f(x, y)+o(1)$ as desired.

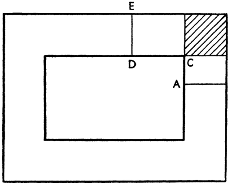


Fig. 7

The second part of the proof of Weierstrass's theorem is as follows. For a suitable $N = N(n)$ we have, for all x, y of R and all ξ, η of R_+,

$$|\mathrm{E} - \Sigma| < n^{-2},$$

where

$$\Sigma = \sum_{m=0}^{N} \frac{[-n\{(\xi-x)^2+(\eta-y)^2\}]^m}{m!}.$$

Then

$$\phi_n(x, y) = \Pi + o(1),$$

where $\Pi = \pi^{-1} n \iint_{R_+} \Sigma\,d\xi d\eta$, and is evidently a polynomial in (x, y).

More from Littlewood

A minute I wrote (about 1917) for the Ballistic Office ended with the sentence 'Thus σ should be made as small as possible'. This did not appear in the printed minute. But P. J. Grigg said, 'what is that ?' A speck in a blank space at the end proved to be the tiniest σ I have ever seen (the printers must have scoured London for it).

More from Littlewood

A minute I wrote (about 1917) for the Ballistic Office ended with the sentence ' Thus σ should be made as small as possible '. This did not appear in the printed minute. But P. J. Grigg said, ' what is that ? ' A speck in a blank space at the end proved to be the tiniest σ I have ever seen (the printers must have scoured London for it).

How not to

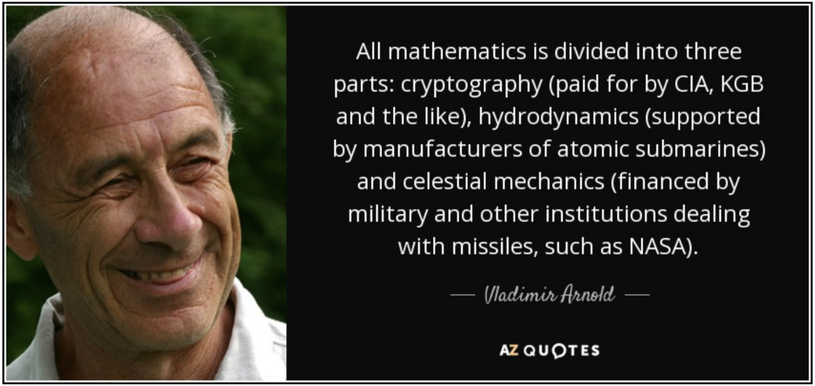
A brilliant but slapdash mathematician once enunciated a theorem in 2 parts, adding : ' part 2, which is trivial, is due to Hardy and Littlewood '.

The trivial part 2 needed to be stated ' for completeness ', and Hardy and Littlewood had similarly needed to state it.

More from Littlewood: a curious acknowledgement

The following idea, a coda to the series, was invented too late (I do not remember by whom), but what *should* have happened is as follows. I wrote a paper for the *Comptes Rendus* which Prof. M. Riesz translated into French for me. At the end there were 3 footnotes. The first read (in French) 'I am greatly indebted to Prof. Riesz for translating the present paper'. The second read 'I am indebted to Prof. Riesz for translating the preceding footnote'. The third read 'I am indebted to Prof. Riesz for translating the preceding footnote', with a suggestion of reflexiveness. Actually I stop legitimately at number 3: however little French I know I am capable of *copying* a French sentence.

From Arnold: famous citations



More from Arnold

Cryptography has generated number theory, algebraic geometry over finite fields, algebra, combinatorics and computers.

More from Arnold

Cryptography has generated number theory, algebraic geometry over finite fields, algebra, combinatorics and computers.

Hydrodynamics procreated complex analysis, partial differential equations, Lie groups and Lie algebra theory, cohomology theory and scientific computing.

More from Arnold

Cryptography has generated number theory, algebraic geometry over finite fields, algebra, combinatorics and computers.

Hydrodynamics procreated complex analysis, partial differential equations, Lie groups and Lie algebra theory, cohomology theory and scientific computing.

Celestial mechanics is the origin of dynamical systems, linear algebra, topology, variational calculus and symplectic geometry.

My favourite citations from Arnold

"The difference between pure and applied mathematics is not scientific but only social. A pure mathematician is paid for uncovering new mathematical facts. An applied mathematician is paid for the solution of quite specific problems."

My favourite citations from Arnold

"The difference between pure and applied mathematics is not scientific but only social. A pure mathematician is paid for uncovering new mathematical facts. An applied mathematician is paid for the solution of quite specific problems."

Example

Columbus began by making a purely applied study, trying to find the way to China, and he was being paid for this. The end of his voyage is reminiscent of a fact of pure mathematics.

My favourite citations from Arnold

"The difference between pure and applied mathematics is not scientific but only social. A pure mathematician is paid for uncovering new mathematical facts. An applied mathematician is paid for the solution of quite specific problems."

Example

Columbus began by making a purely applied study, trying to find the way to China, and he was being paid for this. The end of his voyage is reminiscent of a fact of pure mathematics.

"What is a group? Algebraists teach that this is supposedly a set with two operations that satisfy a load of easily-forgettable axioms..."

My favourite citations from Arnold

"The difference between pure and applied mathematics is not scientific but only social. A pure mathematician is paid for uncovering new mathematical facts. An applied mathematician is paid for the solution of quite specific problems."

Example

Columbus began by making a purely applied study, trying to find the way to China, and he was being paid for this. The end of his voyage is reminiscent of a fact of pure mathematics.

"What is a group? Algebraists teach that this is supposedly a set with two operations that satisfy a load of easily-forgettable axioms..."

"Mathematics is the part of physics where experiments are cheap."

Solution for the puzzle #1

Consider the circle of latitude ℓ_1 of length exactly 10 km near the South Pole. Then the hunter's tent can be anywhere (!) on the circle of latitude m_1 that is 10 km north of ℓ_1.

Solution for the puzzle #1

Consider the circle of latitude ℓ_1 of length exactly 10 km near the South Pole. Then the hunter's tent can be anywhere (!) on the circle of latitude m_1 that is 10 km north of ℓ_1.

Furthermore, for the parallels ℓ_n of length equal to $10/n$ km, one can consider the circles of latitude m_n that are 10 km north of them in the vicinity of the South Pole.

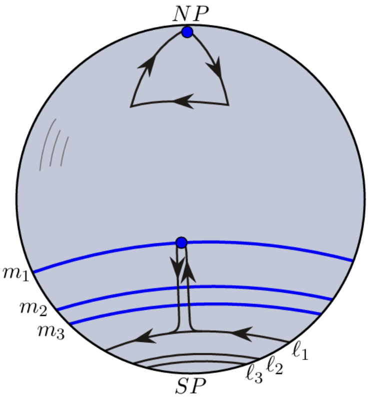
The set of all solutions is huge and not closed!

Solution for the puzzle #1

Consider the circle of latitude ℓ_1 of length exactly 10 km near the South Pole. Then the hunter's tent can be anywhere (!) on the circle of latitude m_1 that is 10 km north of ℓ_1.

Furthermore, for the parallels ℓ_n of length equal to $10/n$ km, one can consider the circles of latitude m_n that are 10 km north of them in the vicinity of the South Pole.

The set of all solutions is huge and not closed!

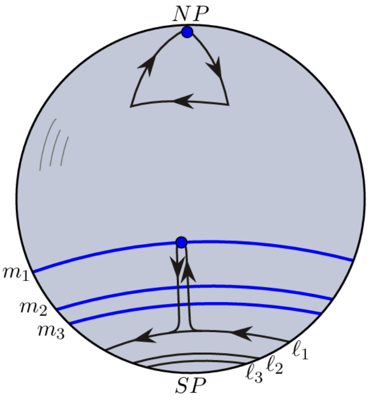


Solution for the puzzle #1

Consider the circle of latitude ℓ_1 of length exactly 10 km near the South Pole. Then the hunter's tent can be anywhere (!) on the circle of latitude m_1 that is 10 km north of ℓ_1.

Furthermore, for the parallels ℓ_n of length equal to $10/n$ km, one can consider the circles of latitude m_n that are 10 km north of them in the vicinity of the South Pole.

The set of all solutions is huge and not closed!



Puzzle #2 is for you!

And maybe... enliven a talk or an article...

Geometric Hydrodynamics in Open Problems   Page 33 of 43   15

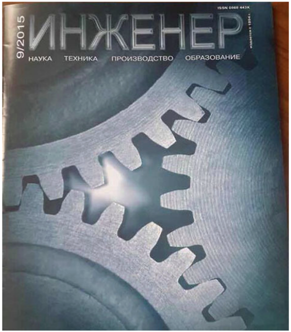


Fig. 4. The 2D direct and inverse cascades together. (Illustration from the cover of Russian magazine "Engineer", where the gears are Education, Science, and Enterprise. No comment)

Literature

– "How to talk Mathematics" by P.R. Halmos, Notices of the AMS 21:3 (1974) , 155-158,

– "How to write mathematics" by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

Also published in "How to Write Mathematics" by N.E.Steenrod, P.R.Halmos, M.M.Schiffer, and J.E.Dieudonne- AMS, 1973, 64pp.

– "A Mathematician's Miscellany" by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

– "On teaching Mathematics" by V.I. Arnold, Russ. Math. Surveys 53(1), 229-236 (1998).

Literature

– “How to talk Mathematics” by P.R. Halmos, Notices of the AMS 21:3 (1974) , 155-158,

– “How to write mathematics” by P.R. Halmos, Enseignement Math. (2) 16 (1970), 123-152.

Also published in “How to Write Mathematics” by N.E.Steenrod, P.R.Halmos, M.M.Schiffer, and J.E.Dieudonne- AMS, 1973, 64pp.

– “A Mathematician’s Miscellany” by J.E. Littlewood, Methuen & Co. Ltd. London (1953).

– “On teaching Mathematics” by V.I. Arnold, Russ. Math. Surveys 53(1), 229-236 (1998).

Thank you!