Is It Possible to Be One-Third Spanish?

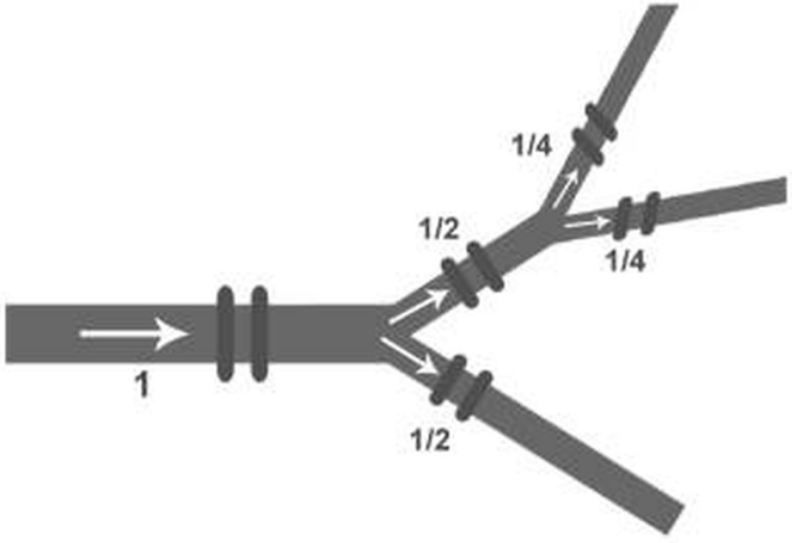
The simple divider problem, proposed by Vladimir Arnold, was studied in connection with designing civil defense drills in the Moscow Metro.

BORIS KHESIN | November 22, 2019 | EL PAÍS — Science / Materia



Vladimir Arnold (1937–2010). Photograph from the original article.

When I was in high school, a mathematics teacher posed a seemingly innocent question to us: “Is it possible to be one-third Spanish?” Clearly, if only one parent is Spanish, their child is one-half Spanish by ancestry. If only one of their four grandparents is Spanish, the grandchild is one-quarter Spanish. But is it possible to be exactly one-third Spanish? Years later, I encountered the same question formulated as a hydrodynamic problem called the “simple divider problem.” The famous mathematician Vladimir Arnold (Odessa, 1937 – Paris, 2010), who was my thesis advisor at the time, studied it in one of his books.



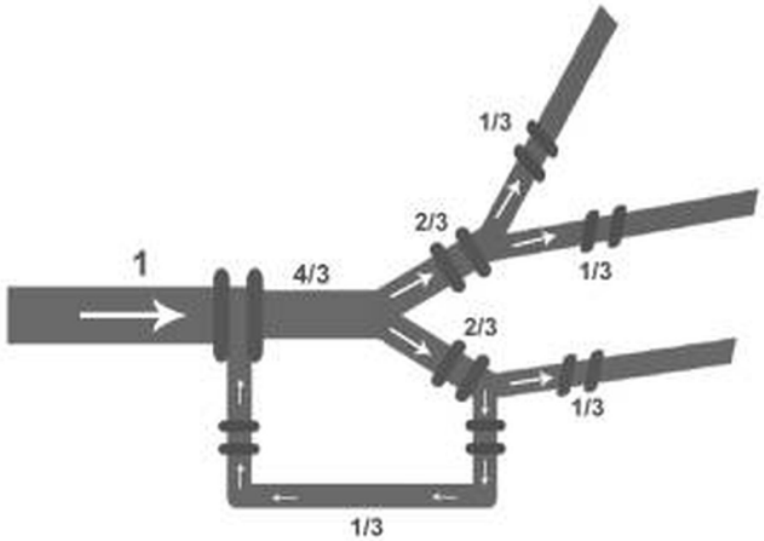
Successive simple dividers split the flow into halves and quarters.

Arnold analyzed the problem from the viewpoint of fluid dynamics: we have a river with a steady flow rate of one cubic meter of water per second, and a device called a simple divider that can split the flow into two equal streams. By applying the divider, we branch the river into two smaller streams, each carrying one-half of the flow; if I use another simple divider on each of these, I obtain streams carrying one-quarter of the original flow. The question is: can a finite number of simple dividers be used to separate off a stream whose flow is one-third of the initial flow?

According to Arnold, this problem arose during civil defense drills in the Moscow Metro, when the organizers wanted to direct a crowd into three identical nuclear shelters. To do so, they needed to divide the flow of people using turnstiles that sent one person to the right and the next to the left (splitting the queue into two equal groups). As with the river, their aim was to divide the original crowd into three equal groups by sending people through successive turnstiles.

Another way to formulate the question is this: three thieves want to divide their loot, raffling off each item in a lottery that gives each of them an equal probability of winning (one-third). But they have only a coin, so they want to do it by tossing heads or tails. Can they organize a fair lottery using this method? Do you see the analogy between this problem and dividing a river into three equal streams?

To solve it, some people suggested an asymptotic solution, that is, dividing the flow infinitely many times so that the flow in one of the small streams converges to one-third.



A feedback loop allows three equal output streams, each carrying one-third of the original flow.

But infinitely many divisions are no use for the thieves, the people in the Metro, the river, or the families. There is another, much simpler answer. Divide the river into four equal streams (using the divider once, and then using another divider on each of the two resulting streams), and route one of them back into the main flow. The remaining three streams are perfectly equal, each with a flow rate equal to one-third of the original. The same idea can be used to obtain any fraction of the flow: simply divide it into more streams

and return as many as necessary to the initial channel.

For the three thieves, this solution would mean following this procedure to raffle off each item of their loot: first, toss the coin twice. If both tosses are heads, the first thief wins the item; if the first is heads and the second tails, the second thief wins; if the first is tails and the second heads, the third thief wins; and if both tosses are tails, start the lottery over again.

This answer does not work for the air-raid shelter problem, because some people would become trapped in a loop and never reach the bunkers. And I do not even dare to spell out the implications involving inbreeding and time travel that this method would have for achieving one-third Spanish ancestry. I prefer to give you a trick answer: Yes, one can be one-third Spanish if both parents are one-third Spanish!

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Spanish translation: Ágata A. Timón. About the section

Coffee and Theorems is a section devoted to mathematics and the environment in which it is created, coordinated by the Institute of Mathematical Sciences (ICMAT). Researchers and members of the institute describe the latest advances in the discipline, explore connections between mathematics and other social and cultural expressions, and remember those who shaped its development and knew how to turn coffee into theorems. The name evokes Hungarian mathematician Alfréd Rényi's definition: "A mathematician is a machine for turning coffee into theorems."

Edited and coordinated by Ágata Timón (ICMAT).

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