EL PAÍS

Coffee and theorems

Do you really understand Earth's cartography?

A famous children's problem about the location of a peculiar

A girl looks at a globe in Copenhagen.

It is not possible to have a perfect coordinate system on curved surfaces. Even on a sphere, any coordinate system we choose has its own peculiarities , as we can see on a map of the Earth. This is the basis of a famous children's puzzle about a hunter who sets up his tent at a point on the planet. According to the puzzle, he leaves his tent and travels 10 km strictly south, 10 km west, and finally 10 km north; at the end of the walk he is back at his camp. Where could his tent be? In some versions it is also mentioned that the hunter saw a bear on his way, and wondered about its color .

As many readers might be thinking, one solution is the North Pole (and the animal's color will be white, since it is a polar bear). However, this of latitude m_1 that is 10 km north of l_1. Follow the path on Fig. 1. Now, instead, we could ask about the color of [the penguins that the hunter will encounter](#) !

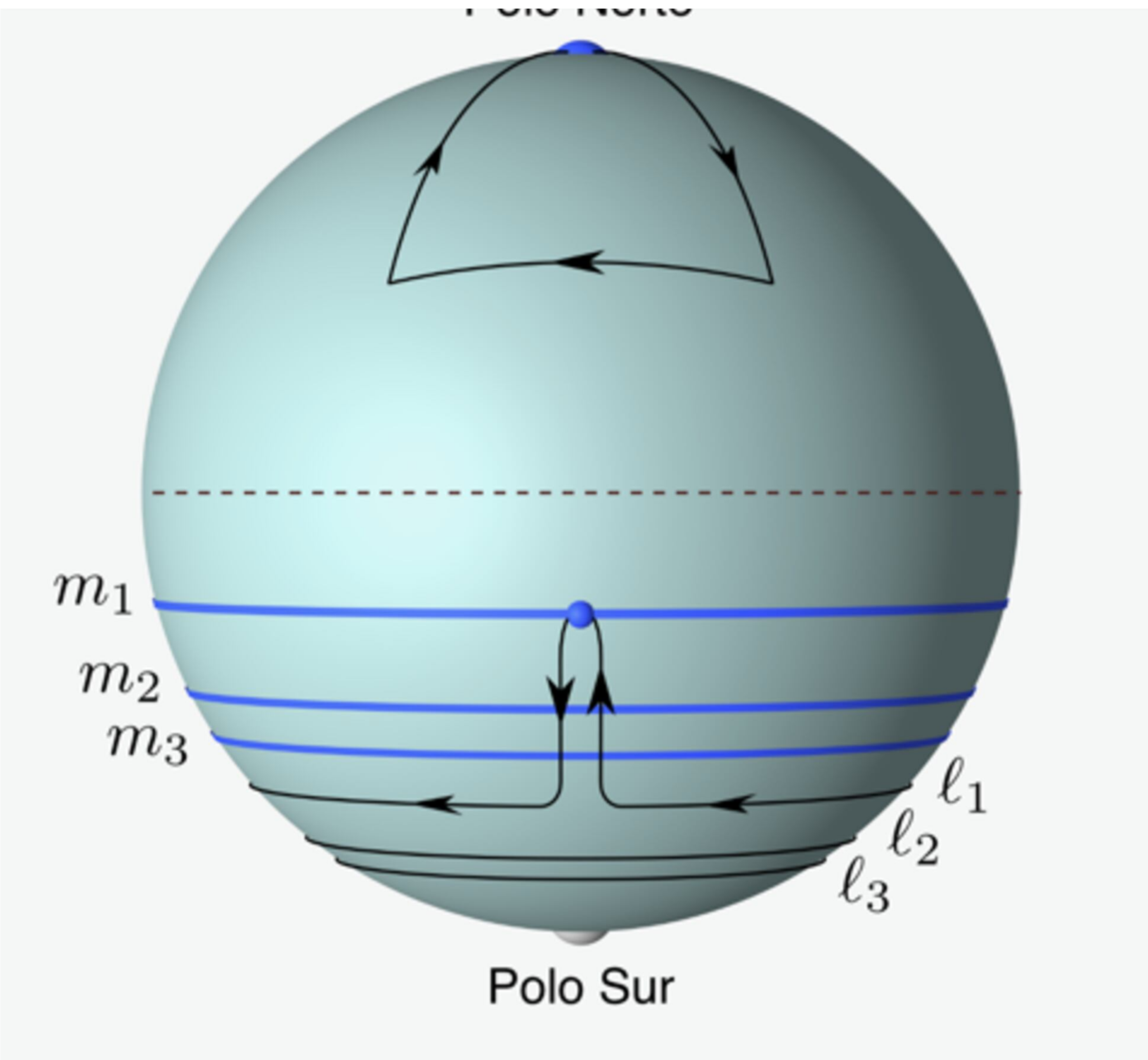

Figure 1
KLAS MODIN

But it does not end there; in the southern hemisphere too, another solution appears by taking a parallel l_2 , 5 km long, and then a parallel m_2 that is 10 km north of it, and proceeding in the same way (in this case, completing two full circles along l_2). In general, the hunter could pitch the South Pole, and complete the indicated path.

We can pose a similar problem, but with a longer path. Now the hunter walks 10 km south, then 10 km west, then 10 km strictly north, and finally 10 km east. When he finishes his walk he is back at his tent. Where is it located?

If the hunter were a flat-earther, he could start anywhere, move along a flat square, and return to the original point. But on Earth, the journey is made along meridians and parallels, and although the hunter covers the same segments when moving north and south on the meridians, he could lose his tent on the way back, because of the different lengths of the circles of the parallels when moving east and west. One solution would involve placing the tent at a specific location anywhere on the circle that is 5 km north of the equator, so that the journey would take place on two equal circles (see Fig. 2).

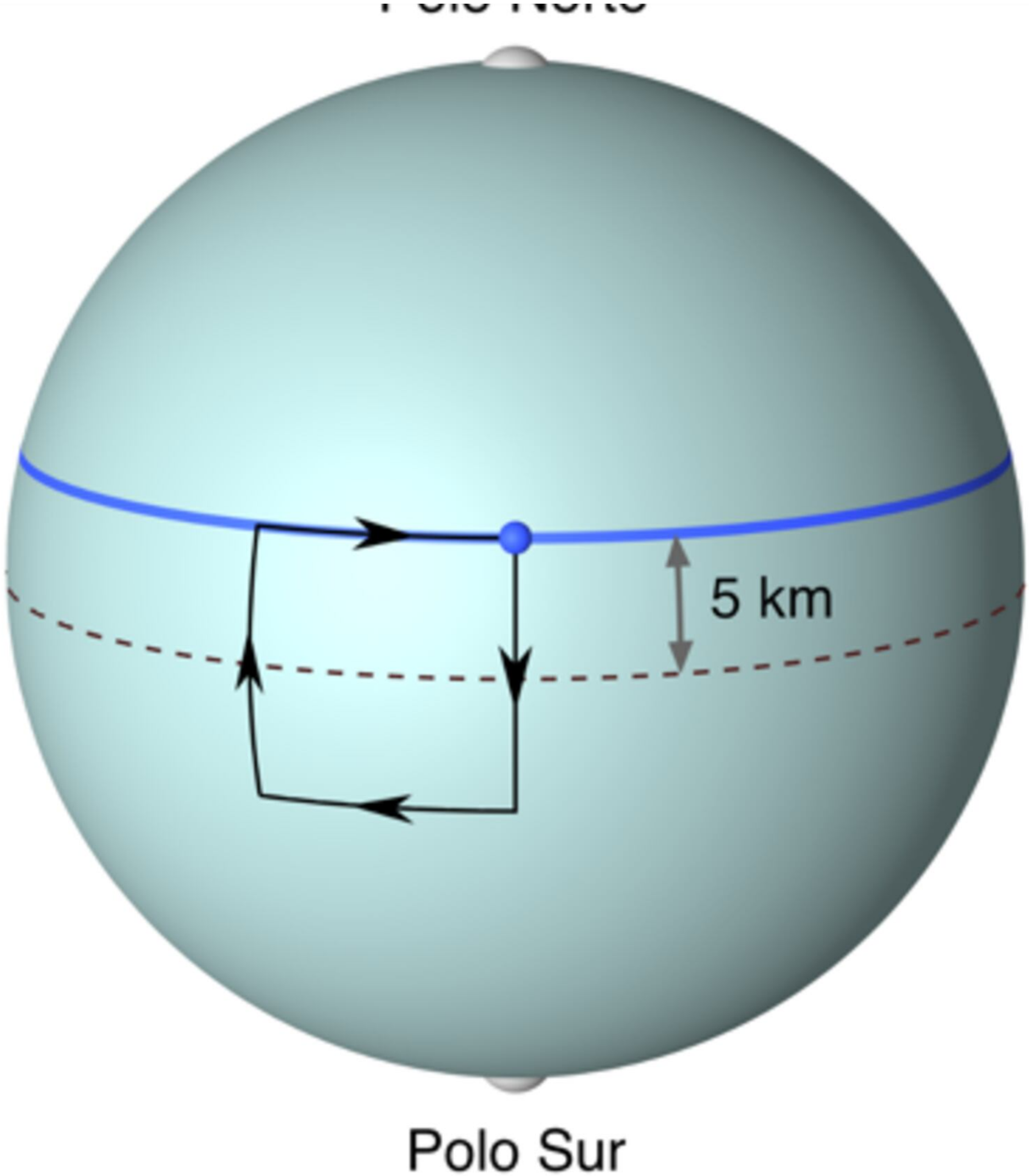

Figure 2
KLAS MODIN

There are other possible locations. By modifying the solutions obtained in the initial puzzle we find a new set of possible paths. The hunter's path will have a keyhole shape (see Fig. 3). We start near the south pole, at any point on the blue parallel which is defined by the following condition. We further, and then return along another meridian, we will reach a point exactly 10 km east (clockwise) of the tent. We know that this blue parallel exists thanks to the intermediate value theorem.

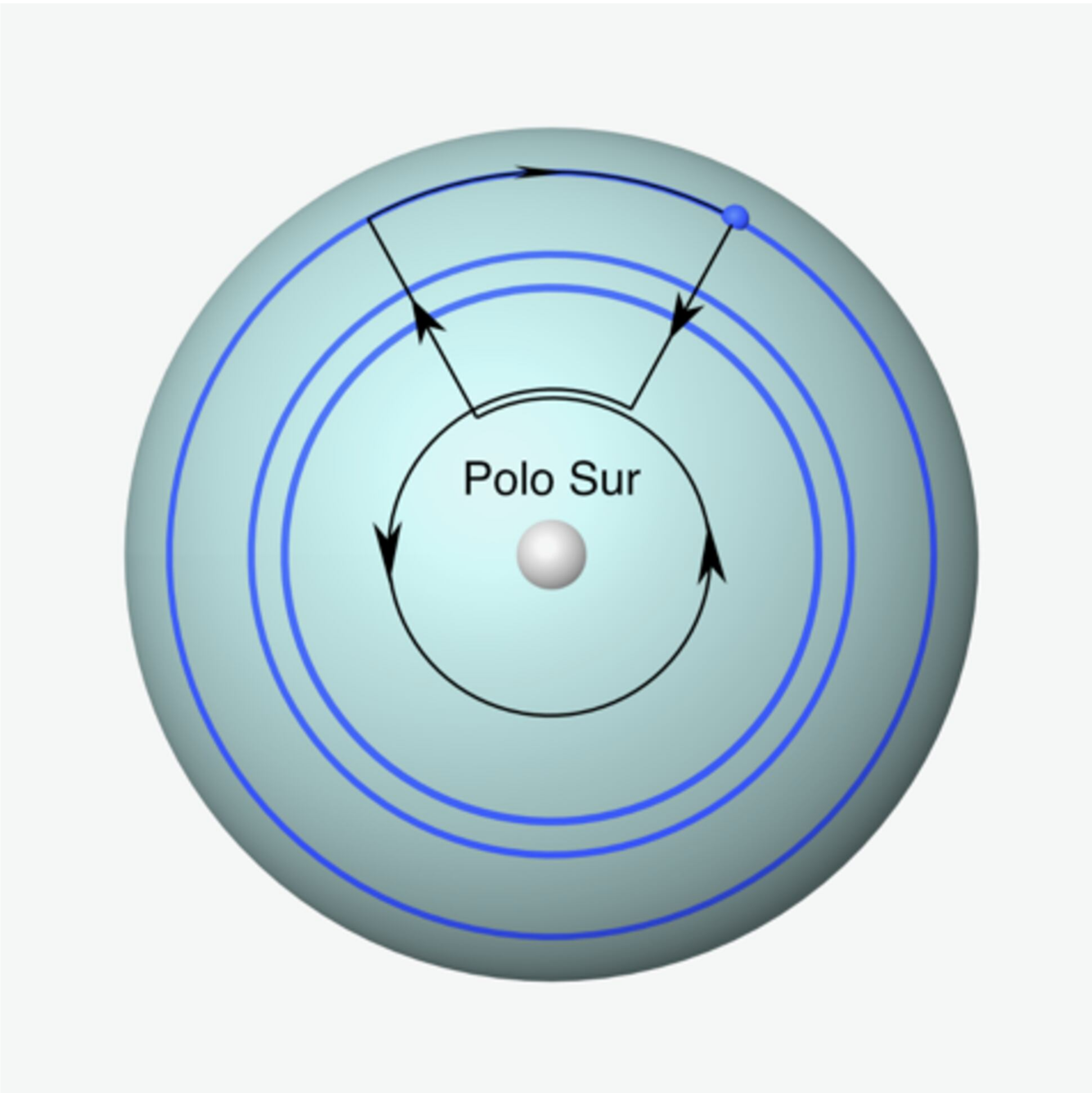

Figure 3

We can find an infinite number of similar solutions: for example, anywhere on the second blue parallel, a little closer to the South Pole, so that when the hunter reaches the short parallel of the circumference a little less than 5 km away, he travels westwards two full circles and a little further along it, before returning along a meridian to the second blue parallel. Similarly, the hunter could pitch his tent anywhere on the infinite number of parallels that have associated corresponding short parallels in the vicinity of the South Pole, and satisfy the above condition. But, in addition, since the problem is symmetrical, we find a similar set of solutions near the North Pole. And these are all the options. Nowhere else is it possible to return to the tent by following the given instructions.

Finally, let's return to the question of what animal the hunter has a chance of finding along the way. In the "polar sets" of solutions, the tent is too close to the North and South Poles to find any animals there, be they bears or penguins. However, in the quasi-equatorial solution, the hunter might find a Galapagos penguin !

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Café y Teoremas *is a section dedicated to mathematics and the environment in which it is created, coordinated by the Institute of Mathematical Sciences (ICMAT), in which researchers and members of the center describe the latest advances in this discipline, share meeting points between mathematics and other social and cultural expressions and remember those who marked its development and knew how to transform coffee into theorems. The name evokes the definition of the Hungarian mathematician Alfred Rényi: "A mathematician is a machine that transforms coffee into theorems."*