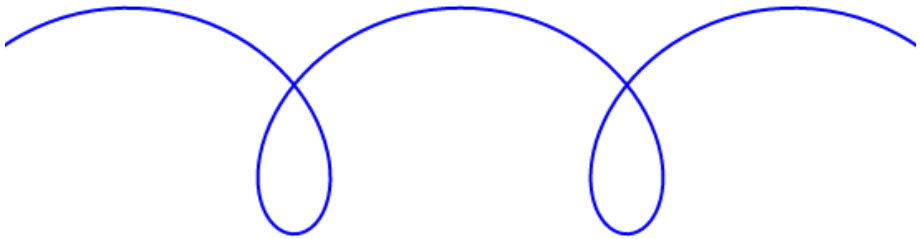
# MAT137

- Problem set 3 due next Wednesday
- Today's Topic: Differentiation, related rates and inverses
- **Watch 4.3-4.8, 5.1-5.4 before next Wednesday**
  Watch 5.2-5.12 before next Friday

# Warm up

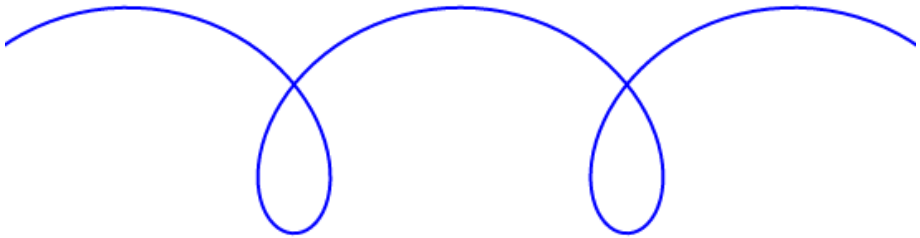
A worm is crawling accross the table. The path of the worm looks something like this:



**True or False?**

The position of the worm in terms of time is a function.

# Worm function



A worm is crawling across the table.
For any time $t$, let $f(t)$ be the position of the worm.
This defines a function $f$.

1. What is the domain of $f$?
2. What is the codomain of $f$?
3. What is the range of $f$?

# Lake ripple

We drop a pebble into a lake. It produces a circular ripple. When the radius is 2 meters and is increasing at a rate of $10cm/s$, at what rate is the area increasing?

# Sliding ladder

A ten-meter long ladder is leaning against a vertical wall and sliding. The top end of the ladder is 8 meters high and sliding down at a rate of 1 meter per second. At which rate is the bottom end sliding?

# Sleepy ants

**Note: We did not do this question in class but it is a good exercise.**

Two ants are taking a nap. The first one is resting at the tip of the minute hand of a cuckoo clock, which is 25 cm long. The second one is resting at the tip of the hour hand, which is half the length. At what rate is the distance between the two ants changing at 3:30?

# Warm up

Compute the derivative of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln x$
2. $f(x) = \pi^{\tan x}$
3. $f(x) = \ln\left[e^x + \ln\ln\ln x\right]$

**Reminder:** We know:

- $\dfrac{d}{dx} e^x = e^x$
- $\dfrac{d}{dx} a^x = a^x \ln a$
- $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$

# A different type of logarithm

Calculate the derivative of

$$f(x) = \log_{x+1}(x^2 + 1)$$

*Hint:* If you do not know where to start, remember the definition of logarithm:

$$\log_a b = c \iff a^c = b.$$

# More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

# More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

**What is wrong with this answer?**

$$\begin{aligned}
\ln f(x) &= (\cos x)\ln(\sin x) + (\sin x)(\ln\cos x) \\
\frac{d}{dx}\left[\ln f(x)\right] &= \frac{d}{dx}\left[(\cos x)\ln(\sin x)\right] + \frac{d}{dx}\left[(\sin x)(\ln\cos x)\right] \\
\frac{f'(x)}{f(x)} &= -(\sin x)\ln(\sin x) + (\cos x)\frac{\cos x}{\sin x} \\
&\quad + (\cos x)\ln(\cos x) + (\sin x)\frac{-\sin x}{\cos x}
\end{aligned}$$

$$f'(x) = f(x)\left[-(\sin x)\ln(\sin x) + (\cos x)\ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x}\right]$$

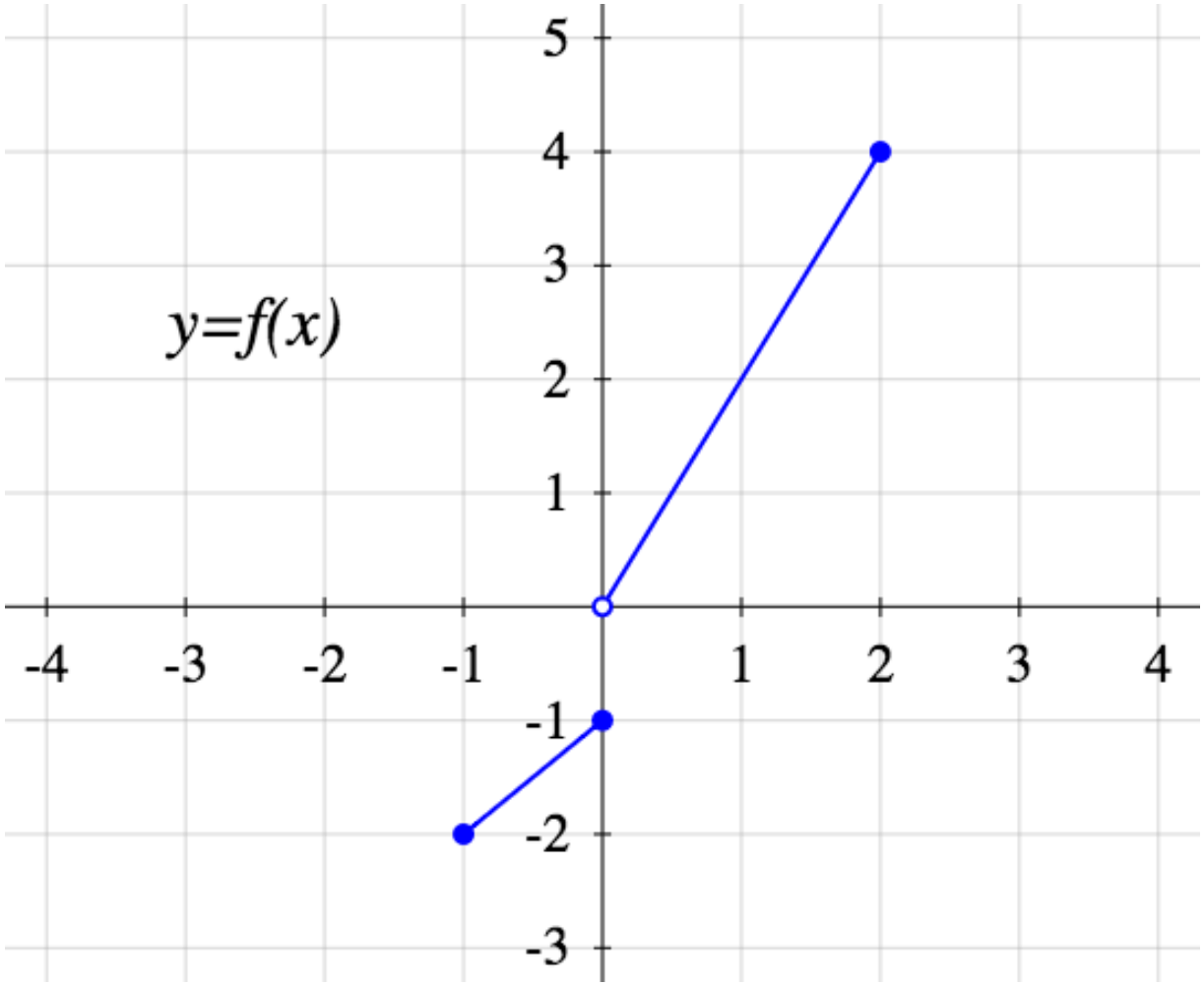
# Hard derivatives made easier

**Note: We did not do this question in class but it is a good exercise.**

Calculate the derivative of

$$h(x) = \sqrt[3]{\frac{\left(\sin^6 x\right)\sqrt{x^7+6x+2}}{3^x\left(x^{10}+2x\right)^{10}}}$$

# Inverse function from a graph




Calculate:

1. $f(2)$
2. $f(0)$
3. $f^{-1}(2)$
4. $f^{-1}(0)$
5. $f^{-1}(-1)$

# Absolute value and inverses

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Find an equation for $h^{-1}(x)$.
3. Sketch the graphs of $h$ and $h^{-1}$.
4. Verify that for every $x \in \boxed{???}$, $h(h^{-1}(t)) = t$, and that for every $x \in \boxed{???}$, $h^{-1}(h(t)) = t$.

# Draw a graph from properties

Sketch the graph of a function $g$ satisfying all the following properties:

1. The domain of $g$ is $\mathbb{R}$.
2. $g$ is continuous everywhere except at $-2$.
3. $g$ is differentiable everywhere except at $-2$ and $1$.
4. $g$ has an inverse function.
5. $g(0) = 2$
6. $g'(0) = 2$
7. $\left(g^{-1}\right)'(-3) = -2$.

# Week 4 survery

Week 4 survery:
https://docs.google.com/forms/d/e/1FAIpQLScFkVz9n5DF-N_Hl997PEWp7y2se8P12Hd8t4FuF6mj4w055Q/viewform