- Tutorials on Monday/Tuesday are cancelled for Term Test 1
- Term Test 1 is on Monday at 6-8pm
  - If you need to write the early sitting, email me ASAP
- TA office hours: 4-6pm today
- Today's Topic: EVT, IVT, and derivatives
- **Watch 3.6-3.12 before next Wednesday**
  Watch 3.13-3.20, 4.1, 4.2 before next Friday

Existence

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1).$
2. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [2, 6]$
3. $f(x) = \dfrac{(e^x + 2)\sin x}{x} - \cos x + 3, \quad I = [-2, 2]$
4. $f(x) = \begin{cases} \frac{(e^x+2)\sin x}{x} - \cos x + 3, & x \in [-2, 0) \cup (0, 2] \\ 5, & x = 0 \end{cases}, \quad I = [-2, 2]$

Definition of maximum

Let f be a function with domain I.
Which one (or ones) of the following is (or are) a definition of "f has a maximum on I"?

1. $\forall x \in I, \exists C \in \mathbb{R}$ s.t. $f(x) \leq C$
2. $\exists C \in I$ s.t. $\forall x \in I, f(x) \leq C$
3. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) \leq C$
4. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) < C$

Can this be proven? (Use IVT)

1. Prove that there exists a time of the day when the hour hand and the minute hand of a clock form an angle of exactly 23 degrees.
2. During a Raptors basketball game, at half time the Raptors have 51 points. Prove that at some point they had exactly 26 points.
3. A baby is born with weight 5kg and height 57cm. Prove that at some point in his life, his height in centimetres will be exactly equal to 10 times his weight in kilograms.
   - Additionally, you know he will be 91 cm tall and weighed 13kg when he is 2 years old

Limits at infinity

Compute:

1. $\lim_{x\to\infty} \left(x^7 - 2x^5 + 11\right)$
2. $\lim_{x\to\infty} \left(x^2 - \sqrt{x^5+1}\right)$
3. $\lim_{x\to\infty} \frac{x^2+11}{x+1}$
4. $\lim_{x\to\infty} \frac{x^2+2x+3}{3x^2+4x+5}$
5. $\lim_{x\to\infty} \frac{x^3+\sqrt{2x^6+1}}{2x^3+\sqrt{x^5+1}}$
6. $\lim_{x\to\infty} \arctan(x)$
7. $\lim_{x\to\infty} \frac{\sin(x)}{x}$

Derivatives from the definition

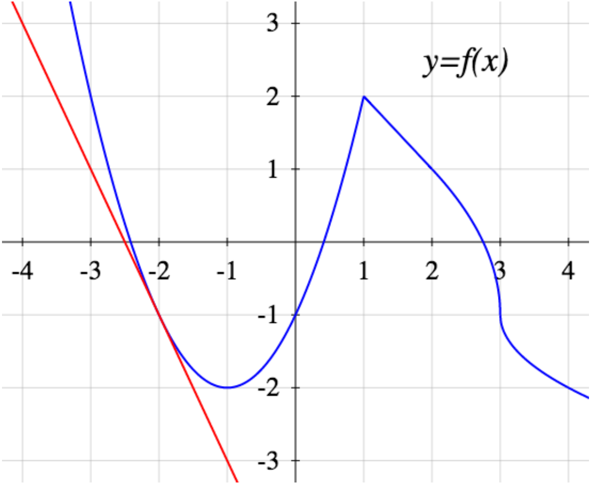
Let

$$g(x) = \frac{2}{\sqrt{x}}$$

Calculate $g'(4)$ directly from the definition of derivative as a limit.

Tangent line from a graph

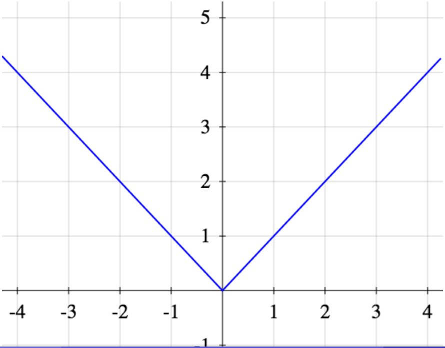
Below is the graph of the function f. Write the equation of the line tangent to it at the point with x–coordinate -2.



Absolute value and tangent lines

At (0,0) the graph of $y = |x|$...

1. ... has one tangent line: $y = 0$
2. ... has one tangent line: $x = 0$
3. ... has two tangent lines $y = x$ and $y = -x$
4. ... has no tangent line



Absolute value and derivatives

Let $h(x) = x|x|$. What is $h'(0)$?

1. It is 0.
2. It does not exist because $|x|$ is not differentiable at 0.
3. It does not exist because the right- and left-limits, when computing the derivative, are different.
4. It does not exist because it has a corner.

Estimations

Let f be a continuous function with domain $\mathbb{R}$.

1. We know $f(4) = 3$ and $f(4.2) = 2.2$.
   Based only on this, give your best estimate for $f(4.1)$.
2. We know $f(4) = 3$ and $f'(4) = 5$.
   Based only on this, give your best estimate for $f(4.1)$.
3. We know $f(4) = 3$ and $f(4.1) = 4$.
   Based only on this, give your best estimate for $f'(4)$.

Estimations – 2

Note: We have not taken up this question yet

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: Consider the values you know for $f(x) = \sqrt[20]{x}$ and its derivative.