- Problem Set 1 is due Wednesday May 15
- Problem Set 2 is available and due **Tuesday** May 21
  - This is so we have time to mark the problem sets before the term test
- Today's Topic: Distance and limits
- **Watch 2.7 - 2.13 before Friday**

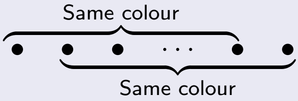
What is wrong with this proof by induction?

Theorem

$\forall N \in \mathbb{Z}$, in every set of N cars, all the cars are of the same colour.

Proof.

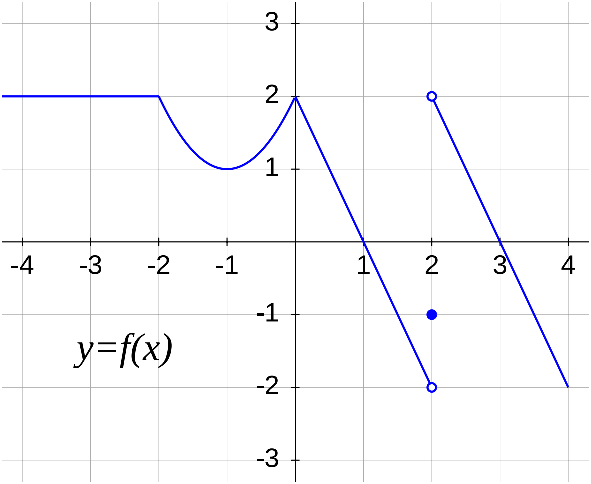
- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for N. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ cars. By induction hypothesis:
  - The first N cars are of the same colour.
  - The last N cars are of the same colour.



Hence the $N + 1$ cars are all of the same colour.

□

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$
3. $\lim_{x \to 2} [f(x)]^2$
4. $\lim_{x \to 0} f(2\cos x)$

Floor

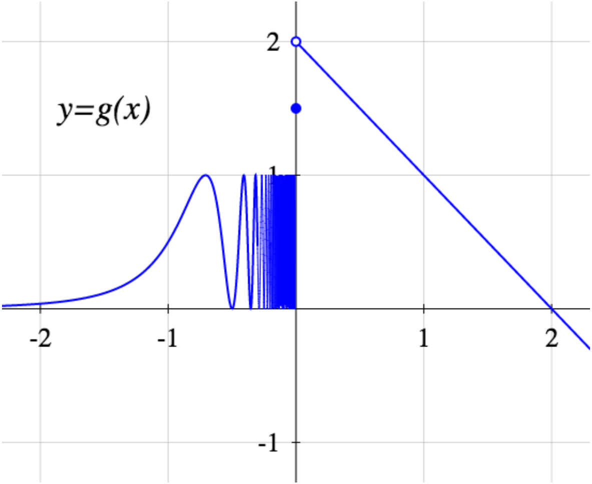
Given a real number x, we defined the *floor of* x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example:

$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$. Then compute:

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

Note: We did not do this question in class



Find the value of

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g(\lfloor x \rfloor)$
4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \lfloor \frac{g(x)}{2} \rfloor$
7. $\lim_{x\to 0^-} g(\lfloor x \rfloor)$

Properties of inequalities

Let $a, b, c \in \mathbb{R}$.
Assume $a < b$. What can we conclude?

1. $a + c < b + c$
2. $a - c < b - c$
3. $ac < bc$
4. $a^2 < b^2$
5. $\frac{1}{a} < \frac{1}{b}$

Properties of absolute value

Let $a, b \in \mathbb{R}$. What can we conclude?

1. $|ab| = |a||b|$
2. $|a + b| = |a| + |b|$

Sets described by distance

Let $a \in \mathbb{R}$. Let $\delta > 0$.
What are the following sets? Describe them in terms of intervals.

1. $A = \{x \in \mathbb{R} \ : \ |x| < \delta\}$
2. $B = \{x \in \mathbb{R} \ : \ |x| > \delta\}$
3. $C = \{x \in \mathbb{R} \ : \ |x - a| < \delta\}$
4. $D = \{x \in \mathbb{R} \ : \ 0 < |x - a| < \delta\}$

Implications

Find *all* values of A, B, and C that make the following implications true

1. $|x-3| < 1 \implies |2x-6| < A$
2. $|x-3| < B \implies |2x-6| < 1$
3. $|x-3| < 1 \implies |x+5| < C$

Write down the formal definition of

$$\lim_{x \to a} f(x) = L.$$

Side limits

We know:

Definition

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \quad \text{and} \quad \lim_{x \to a^-} f(x) = L.$$

Infinite limits

Note: We did not do this question in class

Definition

Let $a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.
Write a formal definition for

$$\lim_{x \to a} f(x) = \infty.$$

Existence

Write down the formal definition of the following statements:

1. $\lim_{x \to a} f(x) = L$

2. $\lim_{x \to a} f(x)$ exists

3. $\lim_{x \to a} f(x)$ does not exist