UNIVERSITY OF TORONTO
FACULTY OF ARTS & SCIENCE

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**Sid Smith Commons:** Tuesday to Friday from 10 am to 12 pm

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# MAT137

- Tutorials start Monday/Tuesday
- Problem Set 1 is due Wednesday May 15
  - You should have received an invitation from Crowdmark to submit
- Today's Topic: Conditionals and proofs
- **Watch videos 2.1 - 2.6 before Wednesday**
  Watch 2.7 - 2.13 before next Friday

# True or False?

Let $x \in \mathbb{R}$.

1. $x > 0 \implies x \geq 0$
2. $x \geq 0 \implies x > 0$
3. If $x > x$, then Kathlyn has a cat.

# Negation again

Write the negation of these statements:

1. If Kathlyn has a cat, then she also has a dog.
2. If someone has a dog, then they don't have a cat.
3. If someone is allergic to pets, then if they have a pet, they have a hypoallergenic dog.

# Cards

Four cards lie on the table in front of you. You know that each card has a letter on one side and a number on the other. At the moment, you can read the symbols $E$, $P$, 3, and 8 on the sides that are up. I tell you:

> *"If a card has a vowel on one side,*
> *then it has an odd number on the other side."*

Which cards do you need to turn over in order to verify whether I am telling the truth or not?

# Maple Syrup

Which of the following statements are equivalent to the statement, "Every Canadian child likes maple syrup"?

1. If a child is Canadian, then they like maple syrup.
2. If a child likes maple syrup, then they are Canadian.
3. If a child does not like maple syrup, then they are not Canadian.
4. If a child is not Canadian, then they like maple syrup.
5. Non-Canadian children do not like maple syrup.
6. If a Canadian does not like maple syrup, then they are an adult.

# 10 minute break

# Definition of odd and even

Write down a formal definition of "odd" and "even" number.

# Definition of odd and even

Write down a formal definition of “odd” and “even” number.

**Definition**

Let $x \in \mathbb{Z}$. We say that $x$ is odd when ...

1. $x = 2a + 1$ ?
2. $\forall a \in \mathbb{Z}, x = 2a + 1$?
3. $\exists a \in \mathbb{Z}$ s.t. $x = 2a + 1$?

# What is wrong with this proof?

**Theorem**

The sum of two odd numbers is even.

**Proof.**

$x = 2a + 1$ odd

$y = 2b + 1$ odd

$x + y = 2n$ even

$2a + 1 + 2b + 1 = 2n$

$2a + 2b + 2 = 2n$

$a + b + 1 = n$ □

# Write a correct proof!

**Theorem**

The sum of two odd numbers is even.

# One-to-one functions

Let $f$ be a function with domain $D$.

$f$ is *one-to-one* means that distinct inputs produce distinct outputs.

**Write a formal definition of “one-to-one”.**

# One-to-one functions

**Definition:** Let $f$ be a function with domain $D$.
$f$ is one-to-one means ...

1. $f(x_1) \neq f(x_2)$
2. $\exists x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
3. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
4. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$
5. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$
6. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$
7. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

# 10 minute break

# Prove a function is one-to-one

**Definition**

Let $f$ be a function with domain $D$.
We say $f$ is one-to-one when

$$\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$$

Equivalently,

$$\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

Prove that $f(x) = -x + 3$ is one-to-one.

# Prove a function is not one-to one

## Definition

Let $f$ be a function with domain $D$.
We say $f$ is one-to-one when

$$\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$$

Equivalently,

$$\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

Prove that $f(x) = x^2$ is not one-to-one.

# Variations on induction 1

Let $S_n$ be a statement that depends on a positive integer $n$.

In each of the following cases, which statements are guaranteed to be true?

1. We have proven:
    - $S_3$ is true.
    - $\forall n \geq 1,\ S_n \text{ is true } \implies S_{n+1} \text{ is true }.$
2. We have proven:
    - $S_1$ is true.
    - $\forall n \geq 3,\ S_n \text{ is true } \implies S_{n+1} \text{ is true }.$
3. We have proven:
    - $S_1$ is true.
    - $\forall n \geq 1,\ S_n \text{ is true } \implies S_{n+3} \text{ is true }.$
4. We have proven:
    - $S_1$ is true.
    - $\forall n \geq 1,\ S_{n+1} \text{ is true } \implies S_n \text{ is true }.$

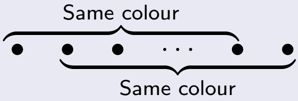
# What is wrong with this proof by induction?

**Theorem**

$\forall N \in \mathbb{Z}$, *in every set of* $N$ *cars, all the cars are of the same colour.*

**Proof.**

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for $N$. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ cars. By induction hypothesis:
  - The first $N$ cars are of the same colour.
  - The last $N$ cars are of the same colour.




Hence the $N + 1$ cars are all of the same colour.

□

# Induction Riddle

There is an island upon which a tribe of 1000 people resides. Their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces). If a tribesperson does discover his or her own eye color, then their religion compels them to leave the island and never return. All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout.

Of the 1000 islanders, it turns out that 100 of them have blue eyes and 900 of them have brown eyes, although the islanders are not aware of these statistics (each of them can of course only see 999 of the 1000 tribespeople).

One day, a blue-eyed foreigner visits the island and wins the complete trust of the tribe. One evening, he addresses the entire tribe to thank them for their hospitality. However, not knowing the customs, the foreigner makes the mistake remarking "how unusual it is to see another blue-eyed person like myself in this region of the world".

What effect, if anything, does this faux pas have on the tribe?

[Hint: What happens if 1 person has blue eyes and 999 people have brown eyes?]

For the discussion on this riddle, see: `https://terrytao.wordpress.com/2008/02/05/the-blue-eyed-islanders-puzzle/`