MAT137

- Problem set 4 due next Wednesday
- If you have a conflict with Test 2 (June 20 at 2pm), you must email me **before June 13**
- Today's Topic: Inverses and local extrema
- **Watch 5.5-5.12 before Friday**

One-to-one functions

1. Write the definition of one-to-one

2. Let f be a function defined by $f(x) = 2x^3 + 7$.
   Prove that f is one-to-one.

3. Let g be a function defined by $g(x) = 2x^2 + 7$.
   Prove that g is not one-to-one.

Composition of one-to-one functions – 2

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Is the following claim TRUE or FALSE? Prove it or give a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one,
THEN f and g are one-to-one.

Definition of arctan

1. Sketch the graph of tan.
2. Prove that tan is not one-to-one.
3. Select the largest interval containing 0 such that the restriction of tan to it is one-to-one. We define arctan as the inverse of this restriction. Let $x, y \in \mathbb{R}$.
$$\arctan y = x \qquad \iff \qquad ???$$
4. What is the domain of arctan? What is the range of arctan? Sketch the graph of arctan.
5. Compute

   5.1 $\arctan(\tan(1))$
   5.2 $\arctan(\tan(3))$
   5.3 $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
   5.4 $\arctan(\tan(-6))$
   5.5 $\tan(\arctan(0))$
   5.6 $\tan(\arctan(10))$

Derivative of arctan

Obtain (and prove) a formula for the derivative of arctan.

Hint: Differentiate the identity

$$\forall t \in \ldots \qquad \tan(\arctan(t)) = t$$

Computation

Note: We did not do this in class but it is a good exercise

Compute the derivative of

$$f(x) = 2x^2 \arctan(x^2) - \ln(x^4 + 1)$$

and simplify it as much as possible.

Draw a graph from properties

Last question from Friday's class

Sketch the graph of a function g satisfying all the following properties:

1. The domain of g is $\mathbb{R}$.
2. g is continuous everywhere except at -2.
3. g is differentiable everywhere except at -2 and 1.
4. g has an inverse function.
5. $g(0) = 2$
6. $g'(0) = 2$
7. $\left(g^{-1}\right)'(-3) = -2$.

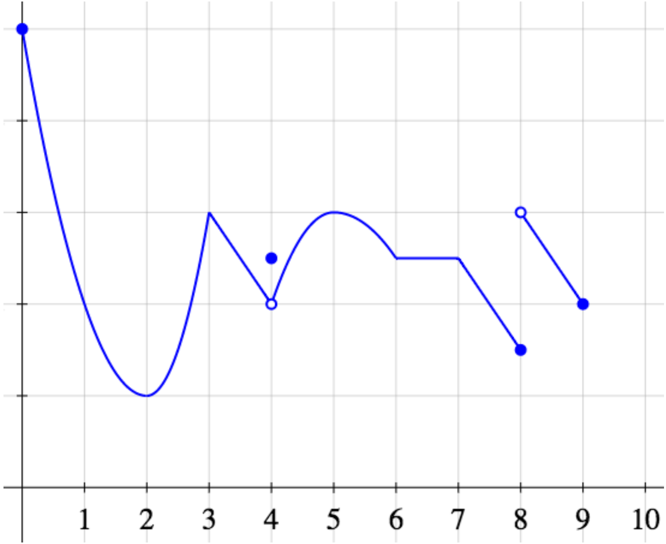
A question from last year's test

Draw the graph of a function f satisfying all of the following:

1. The domain of f is $\mathbb{R}$
2. f is differentiable everywhere
3. The restriction of f to $[0, \infty)$ is one-to-one, and its INVERSE has a vertical tangent line at 2
4. The restriction of f to $(-\infty, 0]$ is one-to-one, and its INVERSE has a derivative of 2 at 2

Definition of local extremum

Find local and global extrema of the function with this graph:



What can you conclude?

We know the following about the function f.

- f has domain $\mathbb{R}$.
- f is continuous
- $f(0) = 0$
- For every $x \in \mathbb{R}$, $f(x) \geq x$.

What can you conclude about $f'(0)$? Prove it.

Hint: Sketch the graph of f. Looking at the graph, make a conjecture.
To prove it, imitate the proof of the Local EVT from Video 5.3.

A sneaky function

Note: We did not do this question in class but it is a good exercise

Construct a function f satisfying all the following properties:

- Domain $f = \mathbb{R}$
- f is continuous
- $f'(0) = 0$
- f does not have a local extremum at 0.
- There isn't an interval centered at 0 on which f is increasing.
- There isn't an interval centered at 0 on which f is decreasing.

Trig extrema

Note: We did not do this question in class but it is a good exercise

Let $f(x) = \dfrac{\sin x}{3 + \cos x}$.

Find the maximum and minimum of f.

How many zeroes?

Note: We did not do this question in class but it is a good exercise

Let

$$f(x) = e^x - \sin x + x^2 + 10x$$

How many zeroes does f have?