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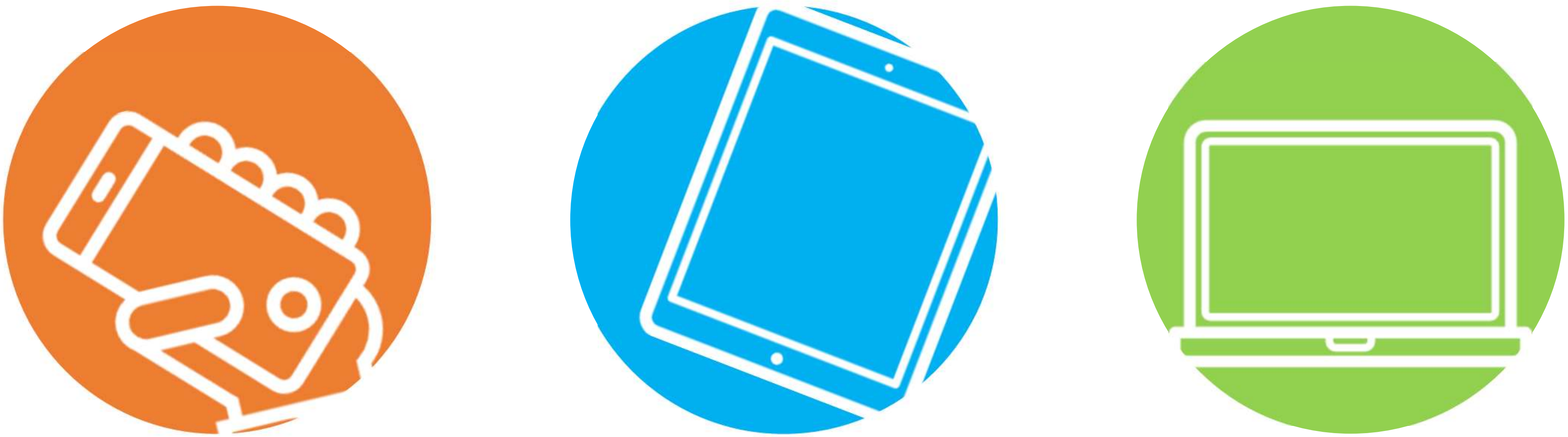

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UNIVERSITY OF TORONTO CENTRE FOR TEACHING SUPPORT & INNOVATION

- Test 2 is at 3:00pm-5:00pm on June 20

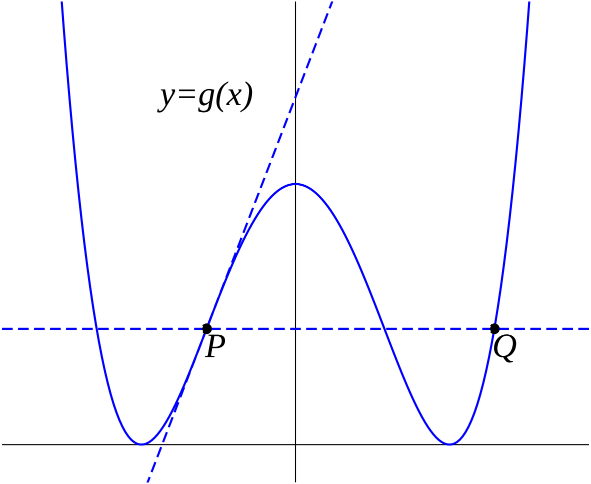
- Classes resume Wednesday July 3
- Tutorials resume Wednesday July 3/Thursday July 4

- Today's Topic: Concavity, asymptotes, graphing

- **Watch 7.1-7.7 before next class**

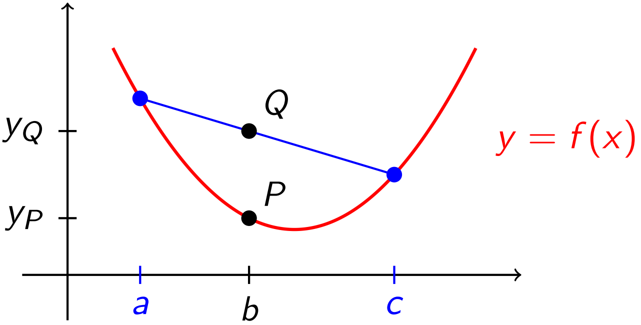
Find the coordinates of P and Q

$$g(x) = x^4 - 6x^2 + 9$$



"Secant segments are above the graph"

Let f be a function defined on an interval I.
In Video 6.11 you learned that an alternative way to define "f is concave up on I" is to say that "the secant segments stay above the graph".



Rewrite this as a precise mathematical statement of the form

$$\text{“}\forall a, b, c \in I, \quad a < b < c \implies \boxed{\text{an inequality involving } f,\ a,\ b,\ c}\text{”}$$

Unusual examples

Construct a function f such that

- the domain of f is at least $(0, \infty)$
- f is continuous and concave up on its domain
- $\lim_{x \to \infty} f(x) = -\infty$

Construct a function g such that

- the domain of g is $\mathbb{R}$
- g is continuous
- g has a local minimum $x = 0$
- g has an inflection point at $x = 0$

Let $f(x) = xe^{-x^2/2}$.

1. Find the intervals where f is increasing or decreasing, and its local extrema.
2. Find the intervals where f is concave up or concave down, and its inflection points.
3. Calculate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.
4. Using this information, sketch the graph of f.

A function with fractional exponents

Note: We did not do this question in class but it is a good exercise

Let $h(x) = \dfrac{x^{2/3}}{(x-1)^{2/3}}$. Its first two derviatives are

$$h'(x) = \frac{-2}{3x^{1/3}(x-1)^{5/3}} \qquad h''(x) = \frac{2(6x-1)}{9x^{4/3}(x-1)^{8/3}}$$

1. Find all asymptotes of h
2. Study the monotonicity of h and local extrema
3. Study the concavity of h and inflection points
4. With this information, sketch the graph of h

Hyperbolic tangent

Note: We did not do this question in class but it is a good exercise

The function tanh, defined by

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}},$$

is called the "hyperbolic tangent".

1. Find its two asymptotes
2. Study its monotonicity
3. Study its concavity
4. With this information, sketch its graph.

Unexpected asymptotes

Find the two asymptotes of the function

$$F(x) = x + \sqrt{x^2 + x}$$

Hint: The behaviour as $x \to \infty$ is very different from $x \to -\infty$.

Slant asymptotes

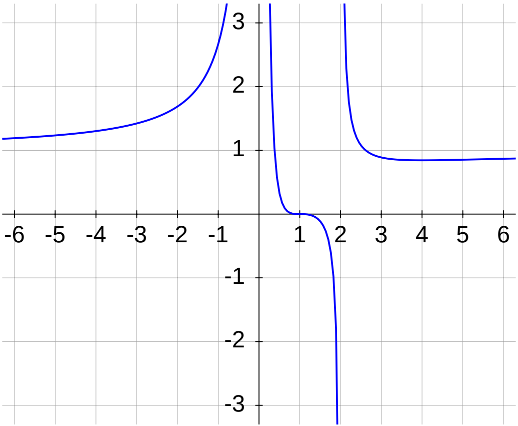
Suppose that a function $f(x)$ has a slant asymptote of $mx + b$ as $x \to \infty$.

1. What can you conclude about $\lim_{x\to\infty} [f(x) - mx]$?
2. What can you conclude about $\lim_{x\to\infty} \frac{f(x)}{x}$?

Backwards graphing

Note: We did not do this question in class but it is a good exercise

This is the graph of $y = R(x)$. R is a rational function (a quotient of polynomials). Find its equation.



A polynomial from 3 points

Note: We did not do this question in class but it is a good exercise

Construct a polynomial that satisfies the following three properties at once:

1. It has an inflection point at $x = 2$
2. It has a a local extremum at $x = 1$
3. It has y-intercept at $y = 1$.

Periodic?

Note: We did not do this question in class but it is a good exercise

Construct a function H that has all the following properties at once:

1. The domain of H is $\mathbb{R}$
2. H is strictly increasing on $\mathbb{R}$
3. H is differentiable on $\mathbb{R}$
4. H' is periodic with period with period 2
5. H' is not constant

A very hard function to graph

The function $G(x) = xe^{1/x}$ is deceiving. To help you out:

$$G'(x) = \frac{x-1}{x}e^{1/x}, \qquad G''(x) = \frac{e^{1/x}}{x^3}$$

1. Carefully study the behaviour as $x \to \pm\infty$.
   You should find an asymptote, but it is not easy.
2. Carefully study the behaviour as $x \to 0^+$ and $x \to 0^-$.
   The two are very different.
3. Use G' to study monotonocity.
4. Use G'' to study concavity.
5. Sketch the graph of G.