MAT137

- Problem set 4 is due today
- Test 2 is at 3:00pm-5:00pm on June 20, **if you have a conflict and want to write the alternate sitting, you must email me by tomorrow**
- Today's Topic: Optimization, L'Hopital's Rule
- **Watch 6.11-6.16 before Friday**

The classic farmer problem

A farmer has $300m$ of fencing and wants to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle. What is the largest area that the field can have?

Distance

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Fire

You hear a scream. You turn around and you see Alfonso is on fire. Literally. Luckily, you are next to a river. Alfonso is 10 meters away from the river and you are 5 meters away from the point P on the river closest to Alfonso. You are carrying an empty bucket. You can run twice as fast with an empty bucket as you can run with a full bucket. How far from the point P should you fill your bucket in order to get to Alfonso with a bucket full of water as fast as possible?

Dominion

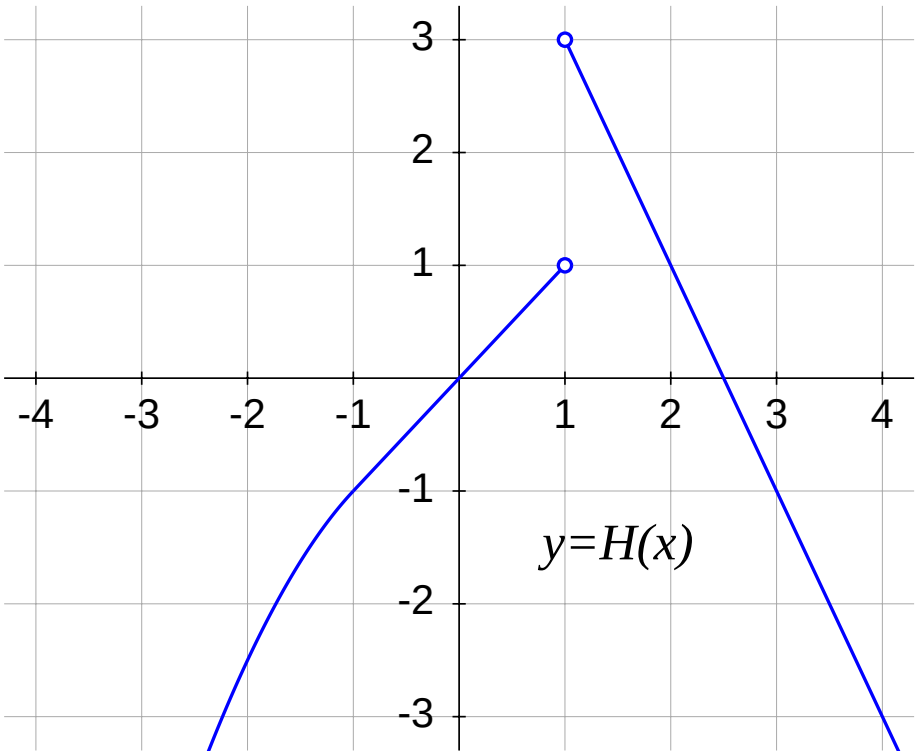
Note: We did not do this question in class but it is a good exercise

Dominion is a board game where, among other things, players buy cards worth victory points. At the end of the game, the player with the most victory points wins. Let's say that you are playing dominion and you are on your last turn. The only cards worth victory points left are called "duchies" and "dukes". A duchy is worth 3 victory points. A duke is worth as many victory points as duchies you have at the end of the game. Each duchy costs 3 coins, and each duke costs 3 coins. You have not bough any duke or duchy yet. If you have N coins and the game allows you to buy as many cards as this affords you, how many dukes and how many duchies should you buy?

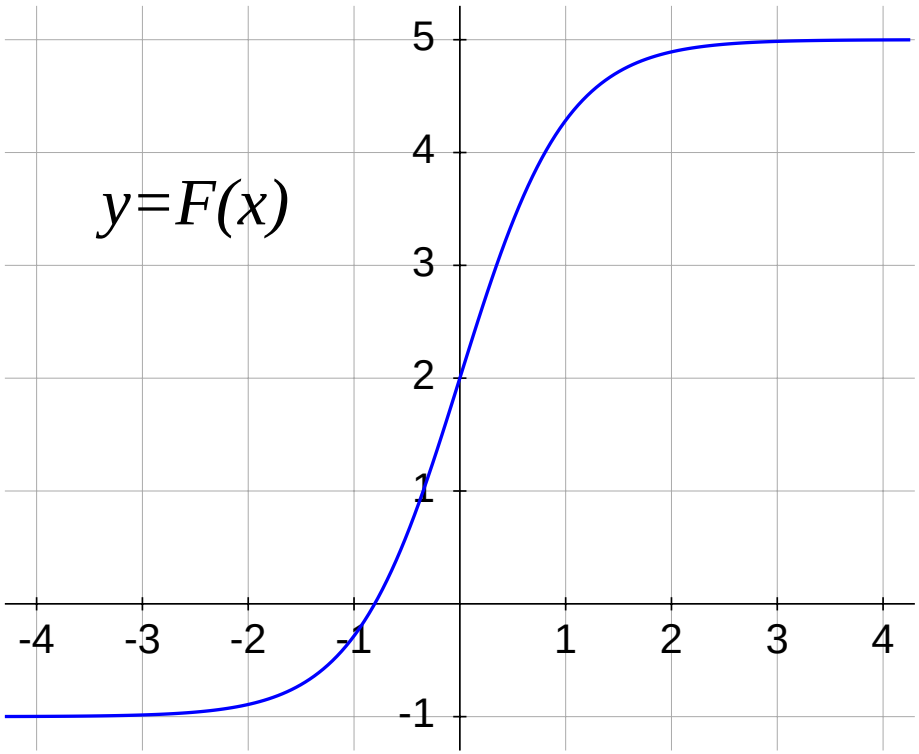
Limits from graphs

Compute:

1. $\lim_{x \to 0} \dfrac{H(x)}{H(2+3x)-1}$



2. $\lim_{x \to 2} \dfrac{F^{-1}(x)}{x-2}$



Computations

Calculate:

1. $\lim_{x \to 2} \dfrac{x^2 + 2x - 6}{x^2 + 3x - 10}$
2. $\lim_{x \to 0} \dfrac{e^{2x^2} - \cos x}{x \sin x}$
3. $\lim_{x \to \infty} \dfrac{x^2}{e^x}$
4. $\lim_{x \to \infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
5. $\lim_{x \to \infty} (\sin x) \left(e^{1/x} - 1 \right)$
6. $\lim_{x \to \infty} x \sin \dfrac{2}{x}$
7. $\lim_{x \to \infty} x \cos \dfrac{2}{x}$
8. $\lim_{x \to 1} \left[(\ln x) \tan \dfrac{\pi x}{2} \right]$

Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This is how you show that $\frac{0}{0}$ is an indeterminate form.

2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.
3. Prove that 1^{∞}, 0^0, and ∞^0 are indeterminate forms. (You will only get $c \geq 0$ this time)

Infinity minus infinity

Compute:

1. $\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$

2. $\lim_{x \to \infty} \left[\ln(x+2) - \ln(3x+4) \right]$

3. $\lim_{x \to 1} \left[\frac{2}{x^2 - 1} - \frac{1}{x - 1} \right]$

4. $\lim_{x \to -\infty} \left[\sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right]$

Exponential indeterminate forms

Note: We did not do this question in class but it is a good exercise

Compute:

1. $\lim_{x \to 0} [1 + 2\sin(3x)]^{4\cot(5x)}$
2. $\lim_{x \to \infty} \left(\frac{x+2}{x-2} \right)^{3x}$
3. $\lim_{x \to 0^+} x^x$
4. $\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$
5. $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$