MAT 247, Winter 2014
Assignment 3
Due Jan 28

January 20, 2014

1. Let V be a vector space and let $T : V \to V$ be a linear operator. Suppose that v is a generalized eigenvector with eigenvalue λ and choose m such that $(T - \lambda I)^m(v) = 0$ but $(T - \lambda I)^{m-1}(v) \neq 0$. Prove that
$$\{v, (T - \lambda I)(v), \cdots, (T - \lambda I)^{m-1}(v)\}$$
are linearly independent.

2. Let V be a complex vector space and let $T : V \to V$ be a linear operator. Prove that $null(T) \oplus im(T) = V$ if and only if $E_0 = K_0$ (every generalized eigenvector with eigenvalue 0 is actually an eigenvector).

3. Let $T : V \to V$ be a linear operator and let $p(x)$ be its minimal polynomial. Prove that T is invertible if and only if x does not divide $p(x)$.

4. Let $T : V \to V$ be a linear operator on a complex vector space. Let $n = \dim V$. We say that V is indecomposable for the action of T if there do not exist T-invariant subspaces W_1, W_2 such that $V = W_1 \oplus W_2$. Prove that V is indecomposable if and only if the minimal polynomial of T is $(x - \lambda)^n$ for some $\lambda \in \mathbb{C}$.

5. Find an operator on a real vector space which is indecomposable, but whose minimal polynomial is not $(x - \lambda)^n$.

6. Let T be a linear operator on a complex vector space. Suppose that $T^m = I$ for some m. Prove that T is diagonalizable.