MAT 247 midterm

Name:

February 15, 2011

1. Let $V, \langle , \rangle$ be an inner product space. Let $W \subset V$ be a subspace.
   (a) Give the definition of $W^\perp$, the orthogonal complement of W.
   (b) Suppose that $W^\perp = V$. Prove that $W = \{0\}$.

2. Consider $\mathbb{R}^3$ with the usual inner product. Let W be the span of $(1,0,0)$ and $(1,1,1)$.

   (a) Perform the Gram-Schmidt process to these vectors to find an orthonormal basis for W.

   (b) Find the orthogonal projection of $(0,0,1)$ onto W.

3. Let V be a real inner product space.

   (a) Given the definition of a self-adjoint linear operator on V.

   (b) Suppose that a linear operator $T : V \to V$ is orthogonally diagonalizable (i.e. there exists an orthonormal basis for V consisting of eigenvectors for T). Show that T is self-adjoint.

4. Let V be an inner product space.

   (a) Give an example of a linear operator $T : V \to V$ such that $\text{null}(T) \neq \text{null}(T^*)$.

   (b) Show that it is not possible to find an example when T is normal.

   (c) Show that for any linear operator $T : V \to V$, $\dim \text{null}(T) = \dim \text{null}(T^*)$.

5. Let $V, \langle , \rangle$ be an inner product space and let $T : V \to V$ be a linear operator. Suppose that for all pairs of vectors $v, w \in V$, $\langle Tv, Tw \rangle = 0$ if and only if $\langle v, w \rangle = 0$ (in other words, T preserves the property of orthogonality). Show that there exists some scalar a such that aT is an isometry.