MAT 247
ASSIGNMENT 8
DUE THURSDAY MARCH 25

(1) Let $V, \langle , \rangle$ be a real inner product space. Let T be a self-adjoint operator on V. On the previous assignment, we saw that T defines a symmetric bilinear form H by the formula $H(v, w) = \langle Tv, w \rangle$.
   (a) Show that $null(T) = rad(H)$.
   (b) Find the signature of H in terms of information about the eigenvalues of T.

(2) Let V, W be two real vector spaces of the same dimension and let H_V, H_W be symmetric bilinear forms on V, W respectively. We say that an invertible linear map $T : V \to W$ is an orthogonal isomorphism if

$$H_V(v_1, v_2) = H_W(Tv_1, Tv_2), \text{ for all } v_1, v_2 \in V.$$

   Prove that there exists an orthogonal isomorphism $T : V \to W$ if and only if the signature of H_V is the same as the signature of H_W.

(3) Let $V = \mathbb{R}^2$. Define a bilinear form H_A on V using the matrix

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}.$$

   What is the signature of H?