ASSIGNMENT 2
DUE THURSDAY OCTOBER 8

(1) Let $X \subset k^n$ be an affine variety. Let $f \in \mathcal{O}(X)$. Let

$$U_f := \{x \in X : f(x) \neq 0\} = X \setminus V(f)$$

the non-vanishing set of the function f.

Show that U_f is an affine variety (hint: not in k^n).

Use this to show that

$$GL_n(k) := \{ \text{ invertible } n \times n \text{ matrices over } k\}$$

is an affine variety.

(2) Exercise II.1 (a)-(d) from Perrin (page 35).

(3) Find an example of a field k (not neccesarily algebraically closed) and a maximal ideal I in $k[x_1, \ldots, x_n]$ such that $k[x_1, \ldots, x_n]/I$ is not isomorphic to k.

(4) Show that the affine variety $\{(a, b) : ab = 1\} \subset k^2$ is not isomorphic to the affine variety k.