ASSIGNMENT 3
DUE THURSDAY FEBRUARY 2

(1) Let F be a field and consider the extension $F \subset F(t)$. The goal of this exercise is to study $Gal(F(t)/F)$.
   (a) Suppose that $a, b, c, d \in F$ with $ad - bc \neq 0$. Prove that the map
   $$f(t) \mapsto f(\frac{at+b}{ct+d})$$
   is an automorphism of $F(t)$.
   (b) Prove that this gives a homomorphism of groups $GL_2(F) \to Gal(F(t)/F)$.
   (c) Prove that this homomorphism is surjective.
   (d) Show that the kernel of this homomorphism consists of multiples of the diagonal matrix.
   (e) Use all this to find the size of $Gal(F(t)/F)$ when $F = \mathbb{F}_q$.
(2) Let $f(x)$ be an irreducible cubic polynomial over a field F. Suppose that K is the splitting field of $f(x)$. What are the possibilities for $Gal(K/F)$?