ASSIGNMENT 1
DUE THURSDAY JANUARY 19

(1) Prove that the field of Laurent series is isomorphic to the fraction field of the field of power series (i.e. show that $k((t)) = Q(k[[t]])$).
(2) Construct a field with 4 elements by adjoining to $\mathbb{F}_2$ the root of an irreducible quadratic polynomial. Find the multiplication table for your field.
(3) Let F be a field of characteristic other than 2. Let D_1, D_2 be elements of F, neither of which is a square in F. Prove that $F(\sqrt{D_1}, \sqrt{D_2})$ is of degree 4 over F if $D_1 D_2$ is not a square in F and is of degree 2 otherwise.
(4) Show that $\mathbb{Q}(\sqrt{2})$ is not isomorphic to $\mathbb{Q}(\sqrt{3})$.
(5) Give an example of a field F and a non-zero map of fields $\phi : F \to F$ which is not an isomorphism. Are there any examples when F is an algebraic extension of $\mathbb{Q}$?
(6) Suppose that α is algebraic over F and that $[F(\alpha) : F] = p$ a prime. Show that for all $1 \leq k < p$, we have $F(\alpha^k) = F(\alpha)$.