

# Welcome to MAT137 - Calculus with proofs!

- Assignment 4 is due on November 26.
- Test 2 begins on December 4.
  
- **Before next class:**
  - **Watch videos 5.10, 5.11**
  - Download next class slides.  
No need to look at them.

## MVT – True or False?

### True or False

Consider  $f(x) = |x|$  on the interval  $[-\frac{1}{2}, 2]$ .

There exists  $c$  in  $(-\frac{1}{2}, 2)$  such that

$$f'(c) = \frac{f(2) - f(-\frac{1}{2})}{2 - (-\frac{1}{2})}$$

## Car race - 1

A driver competes in a race.

Use MVT to prove that at some point during the race the instantaneous velocity of the driver is exactly equal to the average velocity of the driver during the race.

## Car race - 2

Two drivers start a race at the same moment and finish in a tie.

Can you conclude that there was a time in the race (not counting the starting time) when the two drivers had exactly the same speed?

## Car race - Is this proof correct?

### Claim

IF two drivers start a race at the same moment and finish in a tie,  
THEN at some point in the race (not counting the starting time)  
they had exactly the same speed.

### Proof?

- Let  $f(t)$  and  $g(t)$  be the positions of the two cars at time  $t$ .
- Assume the race happens in the interval  $[t_1, t_2]$ . By hypothesis:

$$f(t_1) = g(t_1), \quad f(t_2) = g(t_2).$$

- Using MVT, there exists  $c \in (t_1, t_2)$  such that

$$f'(c) = \frac{f(t_2) - f(t_1)}{t_2 - t_1}, \quad g'(c) = \frac{g(t_2) - g(t_1)}{t_2 - t_1}.$$

- Then  $f'(c) = g'(c)$ .



## Car race - resolution

Two drivers start a race at the same moment and finish in a tie.

Prove that at some point during the race (not counting the starting time) the two drivers had exactly the same speed.