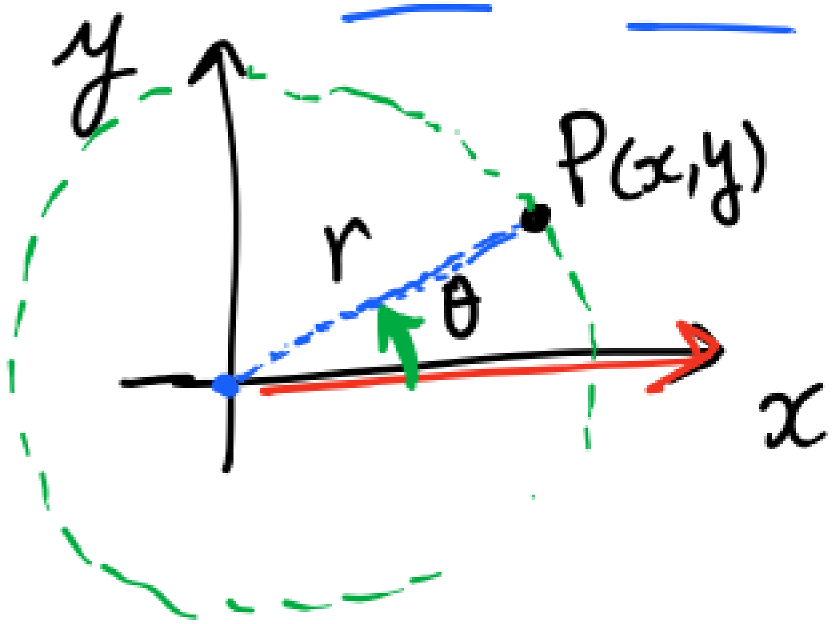
10.3. Polar Coordinates



xy-plane
(cartesian coordinates)
(rectangular)

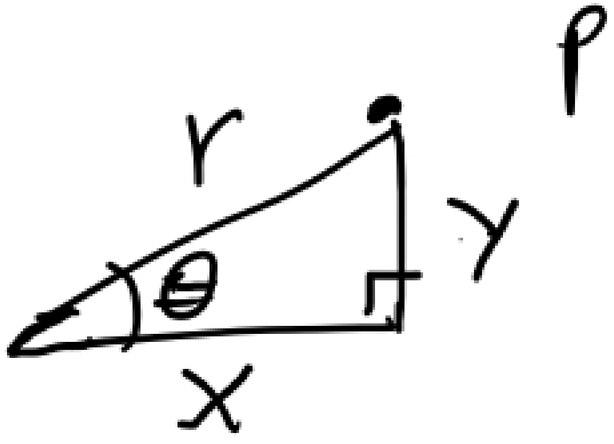
r = distance from P to origin

θ = "polar angle"
angle measured along positive x-axis in a counterclockwise manner.

Polar coordinates for P are (r, θ)

- polar coordinates allow to describe objects that are circular in nature with simpler equations.

Relationship:



$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

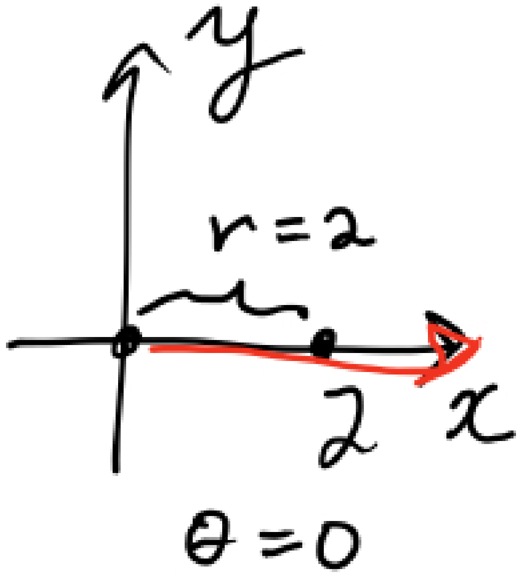
$$\tan\theta = \frac{y}{x}$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

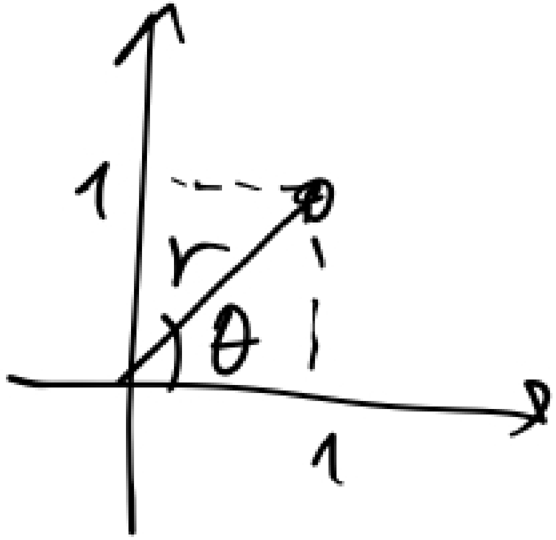
$$r = \sqrt{x^2+y^2}$$
$$\tan\theta = \frac{y}{x}$$

Examples

① $(x,y) = (2,0)$



② $(x,y) = (1,1)$

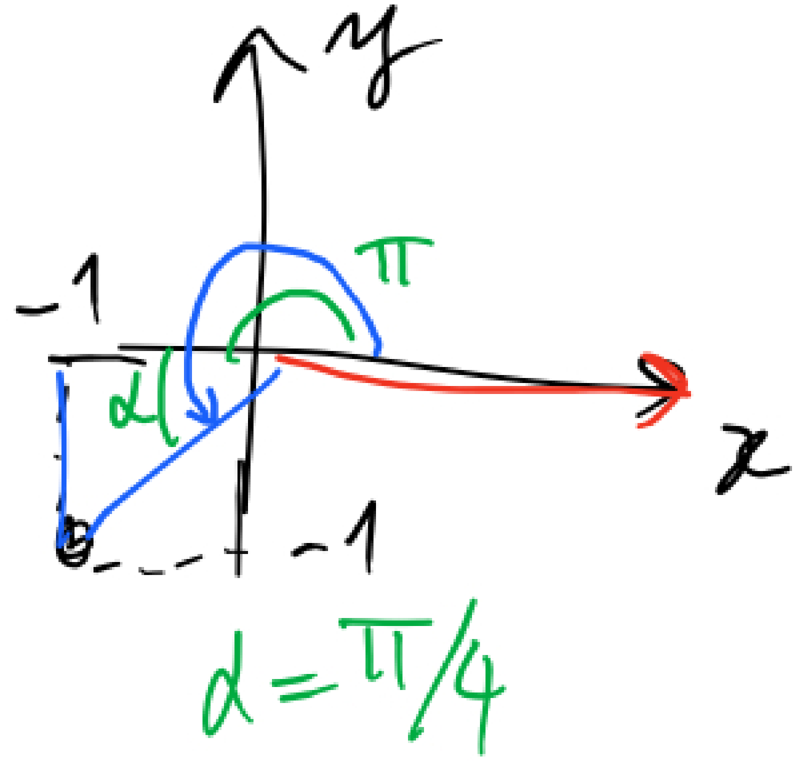


$$r = \sqrt{1^2+1^2} = \sqrt{2}.$$

$$\tan\theta = \frac{1}{1} = 1.$$

$$\theta = \tan^{-1}(1) = \pi/4$$

(3) $(x,y) = (-1,-1)$



$r = \sqrt{(-1)^2+(-1)^2} = \sqrt{2}$

$\tan\theta = \frac{-1}{-1} = 1.$

$\theta = \alpha + \pi = \frac{5}{4}\pi.$

(although $\tan\theta$ is like in Ex. 2!)

Conventions:

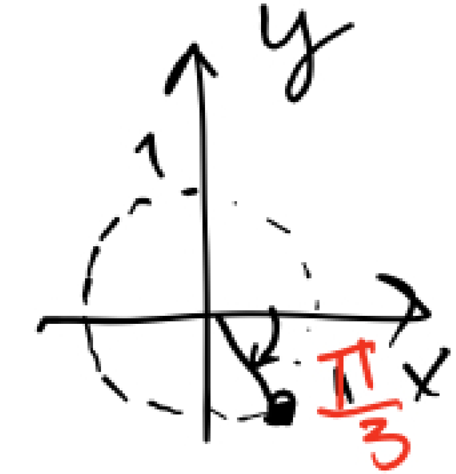
(1) if $r=0$ – this is always the origin.

(2) θ is not unique!

$$(r, \theta) = (r, \theta + 2\pi) = (r, \theta + 4\pi) = \dots$$
$$= (r, \theta - 2\pi) = (r, \theta - 4\pi) = \dots$$

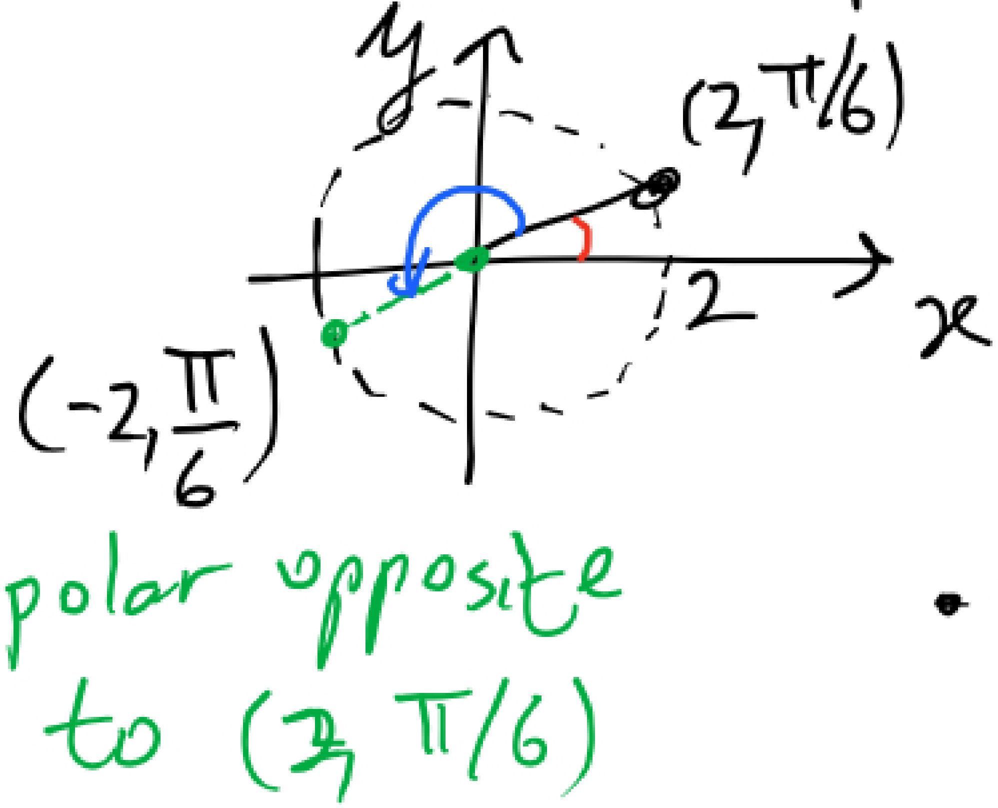
(3) If $\theta < 0$, we measure the angle clockwise

e.g. $(1, -\pi/3)$



(4) <u>$r<0$</u>

Draw $(|r|, \theta)$ and then take its "polar opposite" e.g. $(-2, \pi/6)$



• if $r<0$, then $(r, \theta) =$
$= (|r|, \theta + \pi)$.

Polar curve

Suppose we have an equation $F(r,\theta)=0$.

$F(r,\theta)=0$ - plot all pts that satisfy eq^n

e.g. $r^2+\theta^2-1=0$

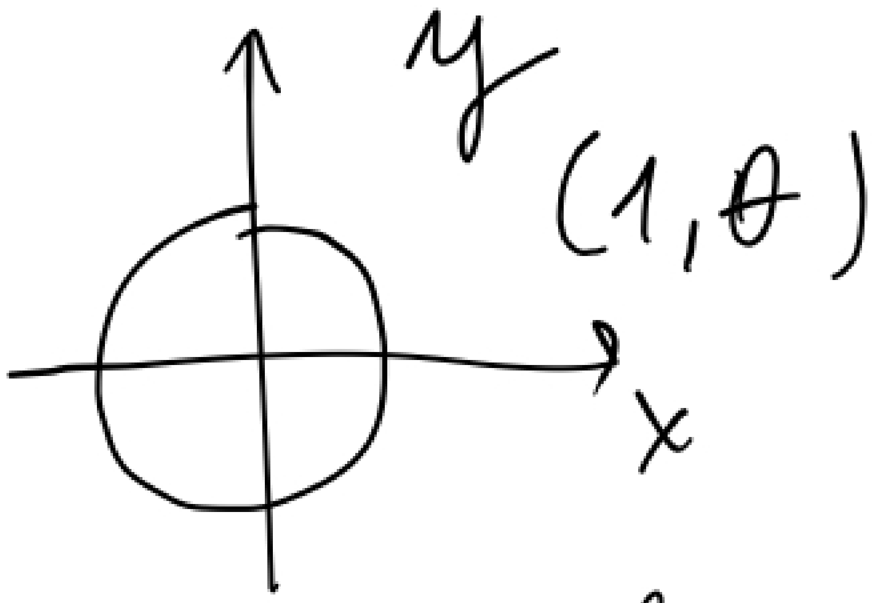
We will looking at curves of the form $r=f(\theta)$.

Examples

1. $r=1$

$\sqrt{x^2+y^2}=1$

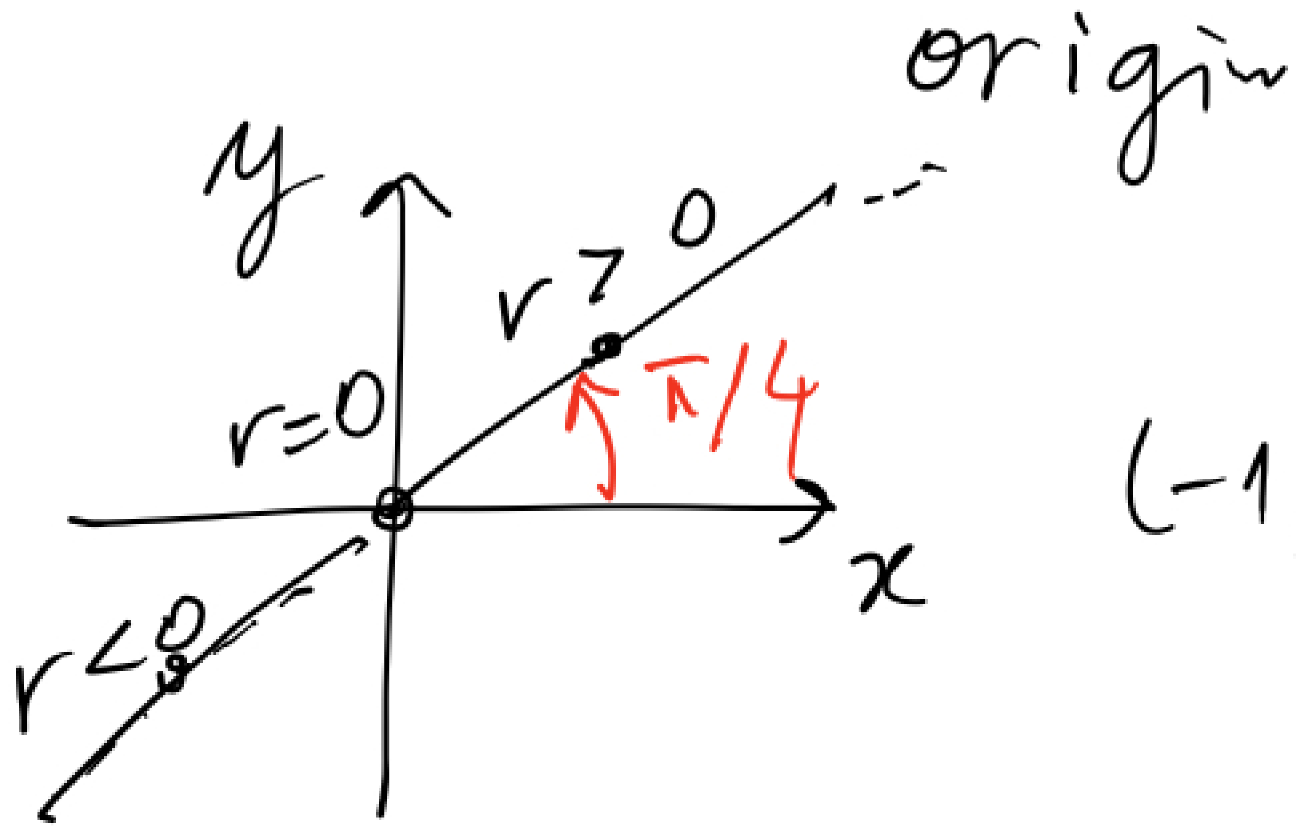
$x^2+y^2=1$.



Unit circle centered at

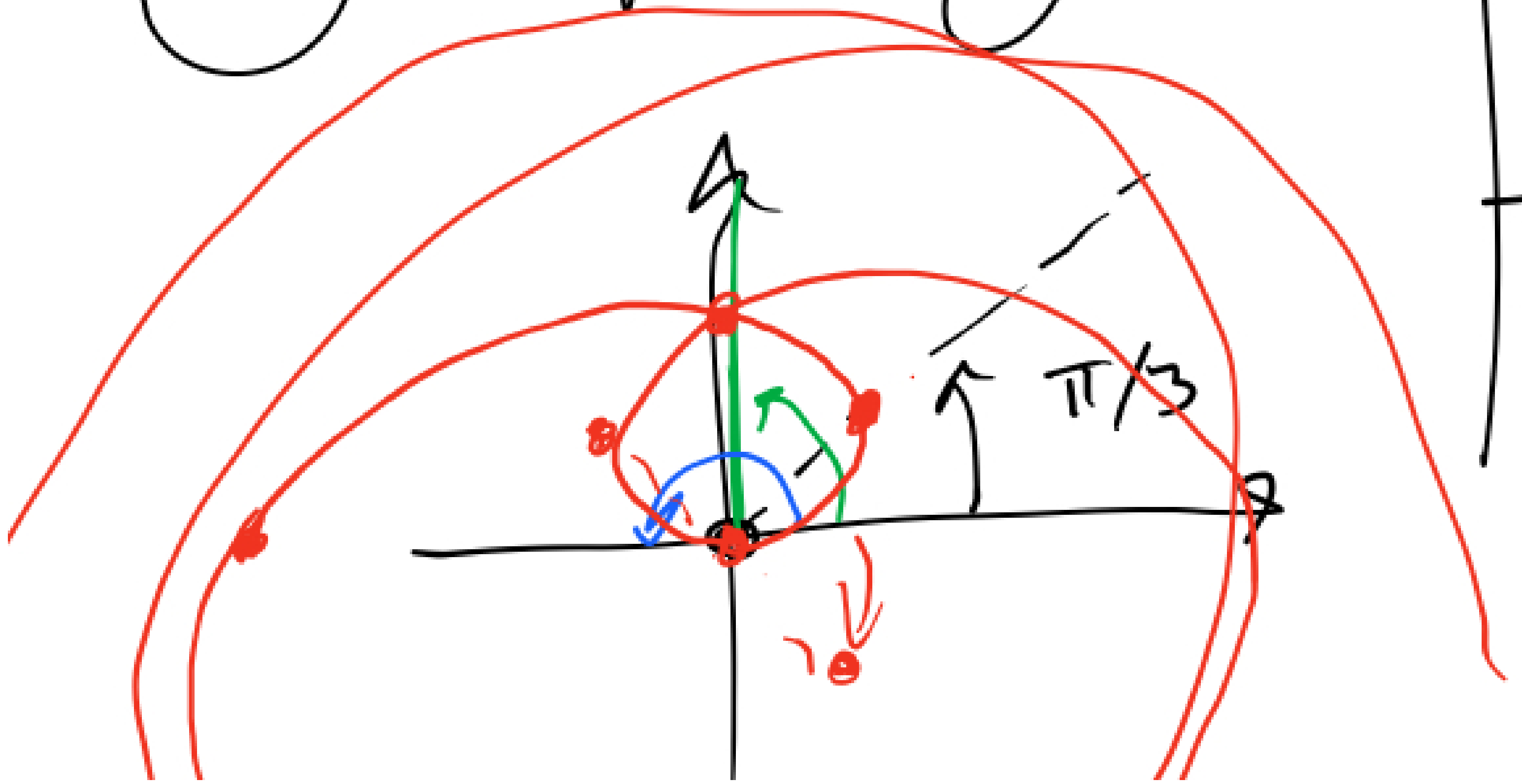
2 $\theta = \pi/4$.

$(r, \pi/4)$



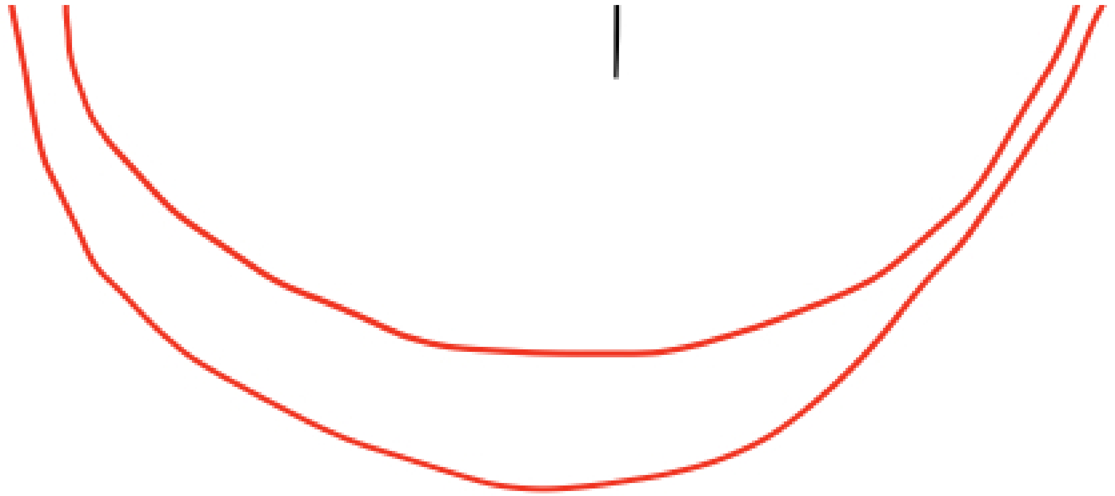
$(-1, \pi/4)$

3 $r = \theta$



θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
r	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π

$\boxed{\tan\sqrt{x^2+y^2} = \frac{y}{x}}$



spiral!

Convention:

$\theta > 0$

for polar curves

(4) $r = 2\cos\theta$

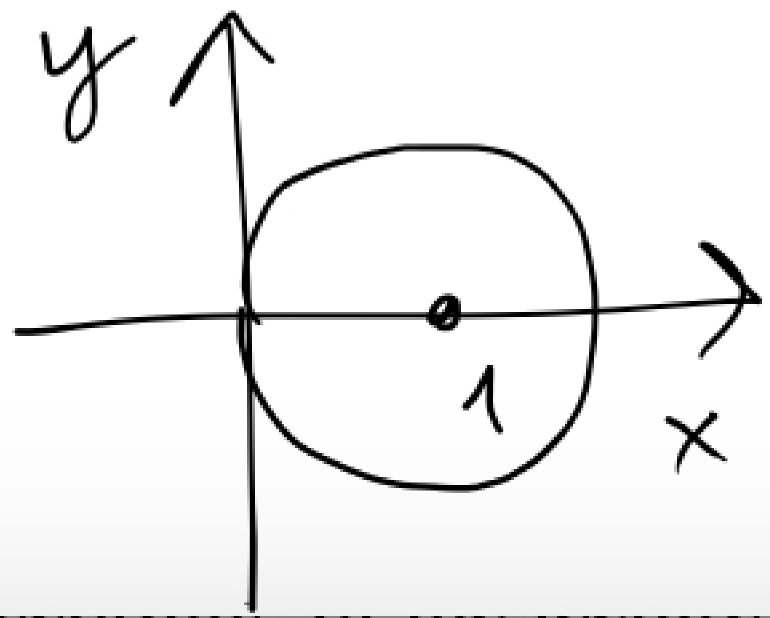
convert to (x,y) coord:

$r = \sqrt{x^2+y^2}$ $\qquad x = r\cos\theta \Rightarrow$

$\Rightarrow \cos\theta = \frac{x}{r}$

$r = 2\cos\theta = 2\frac{x}{r}$

$r^2 = 2x$



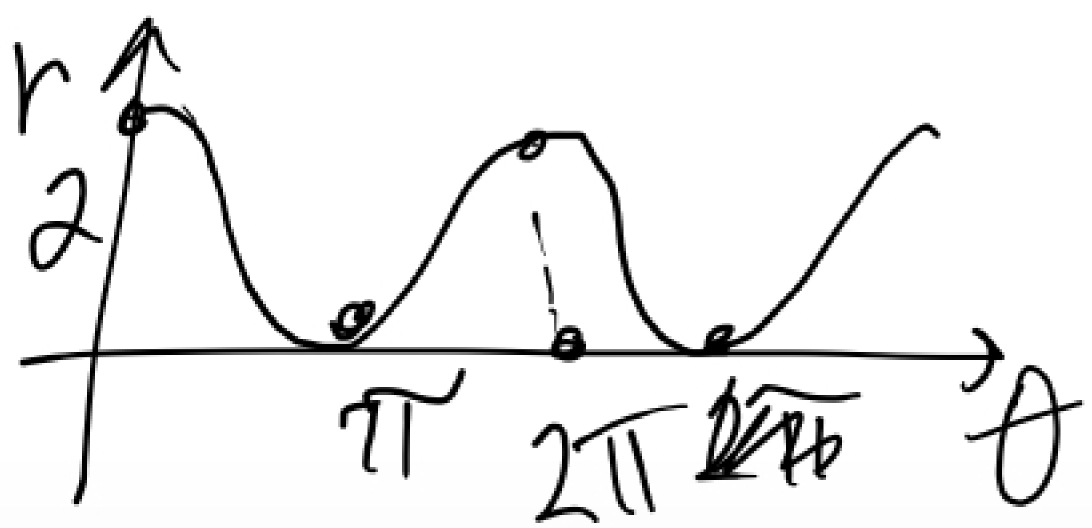
$$x^2+y^2=2x \quad \text{- circle!}$$

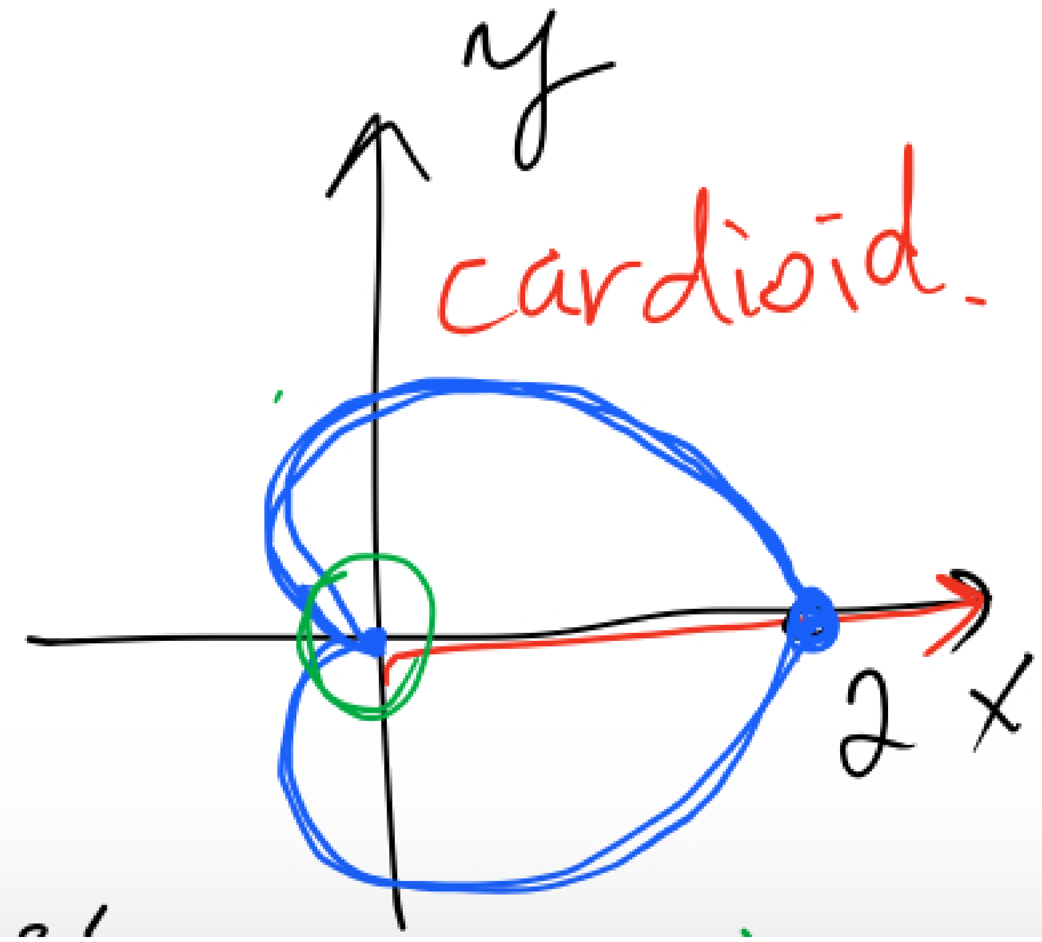
Complete the square: $(x-1)^2+y^2=1$

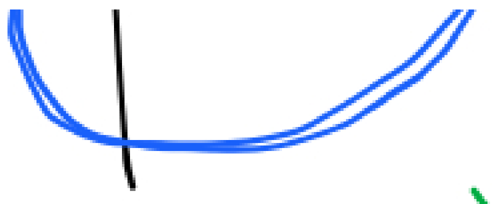
Exercise: $r = 2a\cos\theta$

$r = 2a\sin\theta$

(5) $r = 1+\cos\theta$







For $0 \le \theta \le \pi$, r decreases from 2 to 0.

for $\pi \le \theta \le 2\pi$, r increases from 0 to 2.