Last time:

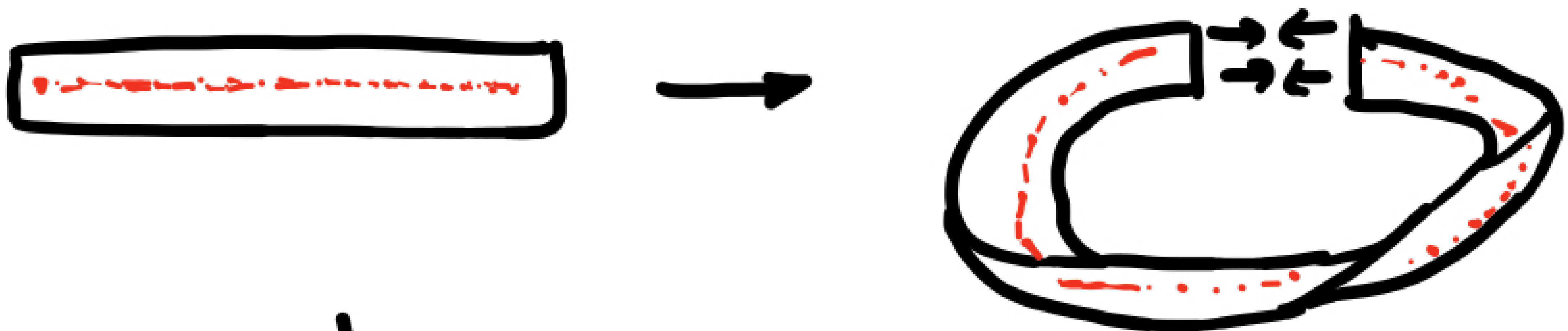
- Surface integral of a function

$$\iint_S f(x,y,z)\,dS$$

To define surface integral of a vector field, we need to talk about ...

# Oriented surfaces

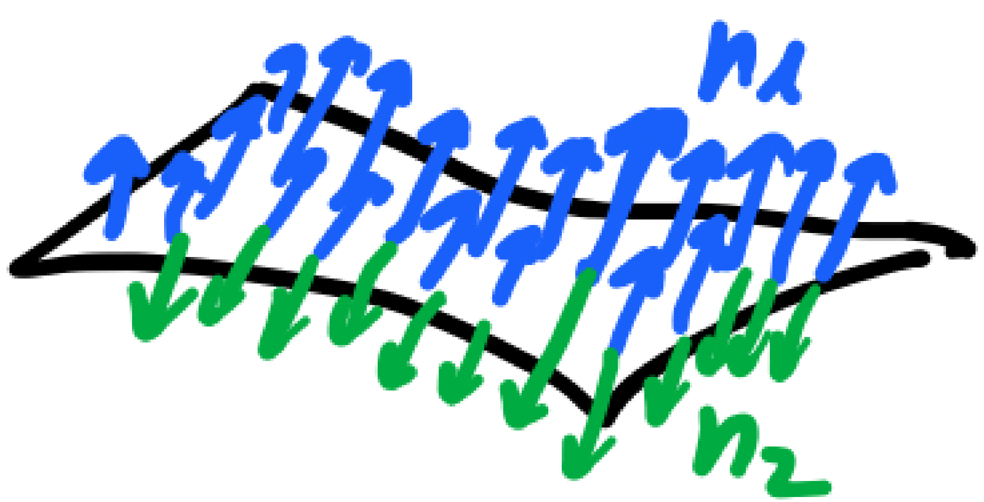
→ Möbius strip – example of non orientable surface.



· it has only one "side"!

We will only consider orientable (two-sided) surfaces.

**Definition** If it is possible to choose a unit normal vector $\vec{n}$ at every point of the surface S such that $\vec{n}$ varies continuously over S, then S is called orientable and a choice of $\vec{n}$ provides S with an orientation.




- There are exactly 2 unit normal vectors at a point
$$\vec{n_1} = -\vec{n_2}$$

- If S is orientable,

then there are 2 choices of orientation.

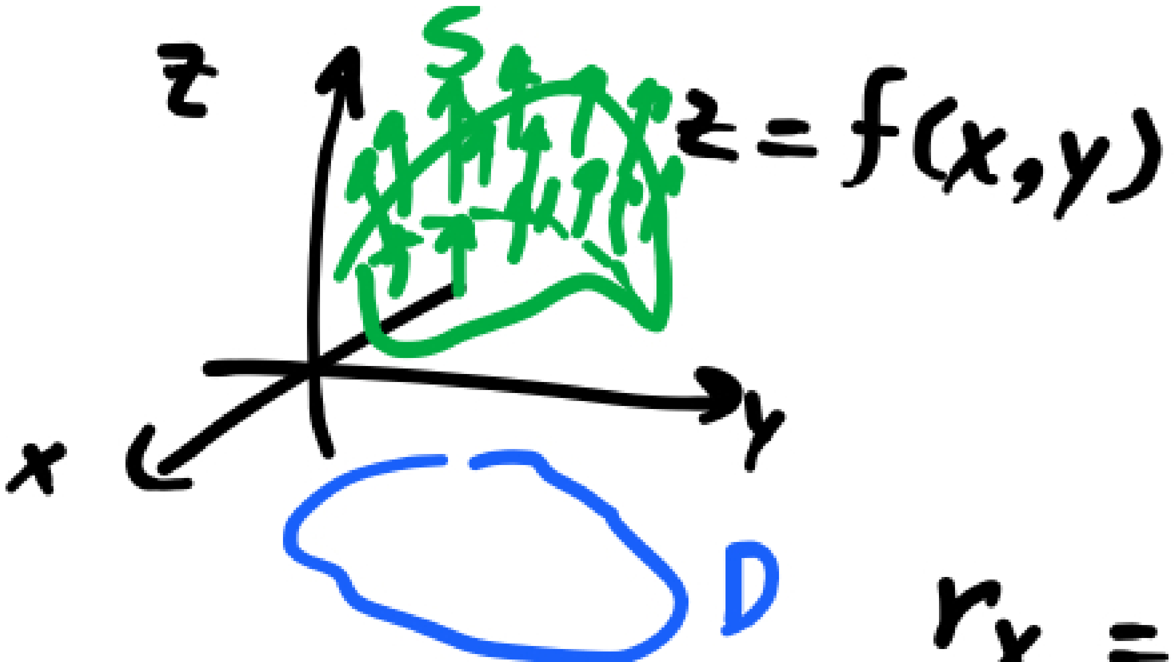
Examples of orientation:

- smooth orientable parametrized $S$

$$\cdot \; \vec{n} = \frac{r_u \times r_v}{\| r_u \times r_v \|}$$

(also you can take the opposite one)

- particular case : graph of a function




$$\begin{cases} x = x \\ y = y \\ z = f(x,y) \end{cases} \qquad (x,y) \in D$$

$$r_x = 1\hat{\imath} + 0\hat{\jmath} + f_x\hat{k}$$

$$r_y = 0\hat{\imath} + 1\hat{\jmath} + f_y\hat{k}$$

$$r_x \times r_y = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & f_x \\ 0 & 1 & f_y \end{vmatrix} = -f_x\hat{\imath} - f_y\hat{\jmath} + \hat{k}.$$

$$\boxed{\vec{n} = \frac{-f_x\hat{\imath} - f_y\hat{\jmath} + \hat{k}}{\sqrt{f_x^2 + f_y^2 + 1}}}$$

<u>Example</u> (sphere) radius $a > 0$

$$\vec{n} = \frac{r_\varphi \times r_\theta}{\| r_\varphi \times r_\theta \|}$$

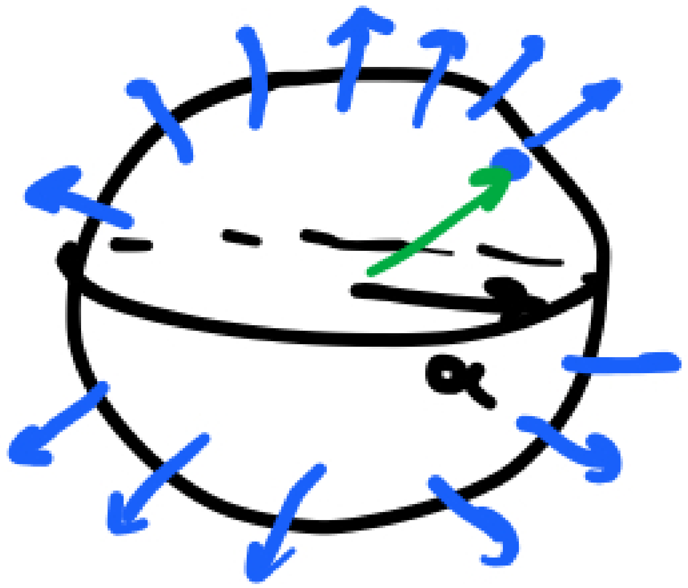
$$\| r_\varphi \times r_\theta \| = a^2 \sin\varphi$$

$$r_\varphi \times r_\theta = a^2 \sin^2\varphi \cos\theta \,\hat{\imath} + a^2 \sin^2\varphi \sin\theta \,\hat{\jmath} + a^2 \sin\varphi \cos\varphi \,\hat{k}$$

(from last time)

$$\vec{n} = \sin\varphi \cos\theta \,\hat{\imath} + \sin\varphi \sin\theta \,\hat{\jmath} + \cos\varphi \,\hat{k} =$$

$$= \frac{1}{a} r(\varphi, \theta).$$




For a closed surface
(boundary of a solid E):

positive orientation – outward looking

negative orientation – inward looking