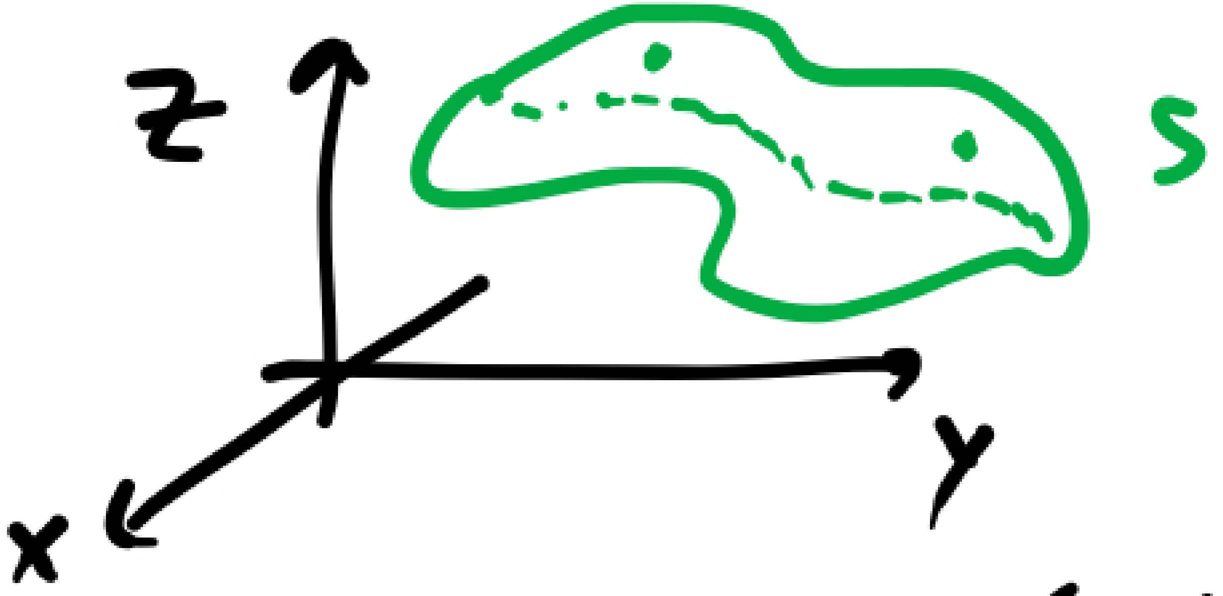
- $\|r_\varphi \times r_\theta\| = a^2 \sin\varphi$

$$A(\text{sphere}) = \iint_D a^2 \sin\varphi \, dA = \int_0^{2\pi} \int_0^{\pi} a^2 \sin\varphi \, d\varphi \, d\theta =$$

$$= a^2 \cdot \int_0^{2\pi} d\theta \cdot \underbrace{\int_0^{\pi} \sin\varphi \, d\varphi}_{=2} = \boxed{4\pi a^2}$$

16.7. Surface Integral



S – smooth surface

$f(x,y,z)$ – continuous function

$$\iint_S f(x,y,z)\,dS = \,?$$

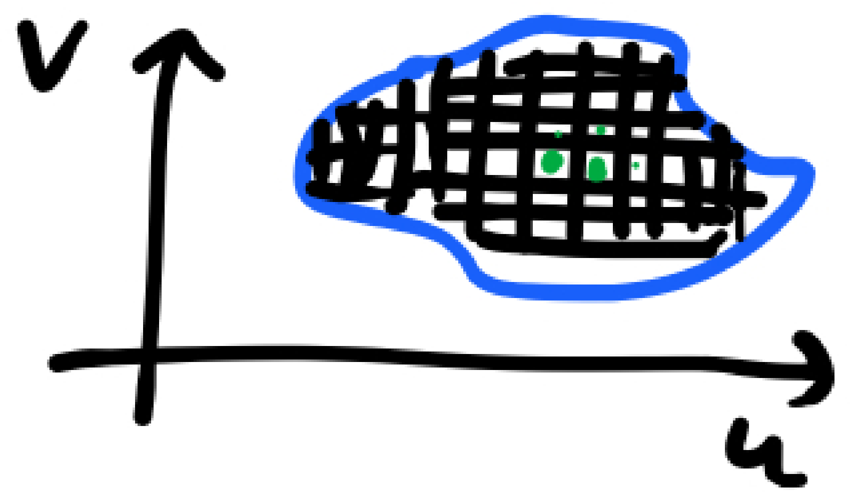
surface integral of a function

- we can think about it as of a generalization of the line integral w.r.t. arclength (+1 dimension)

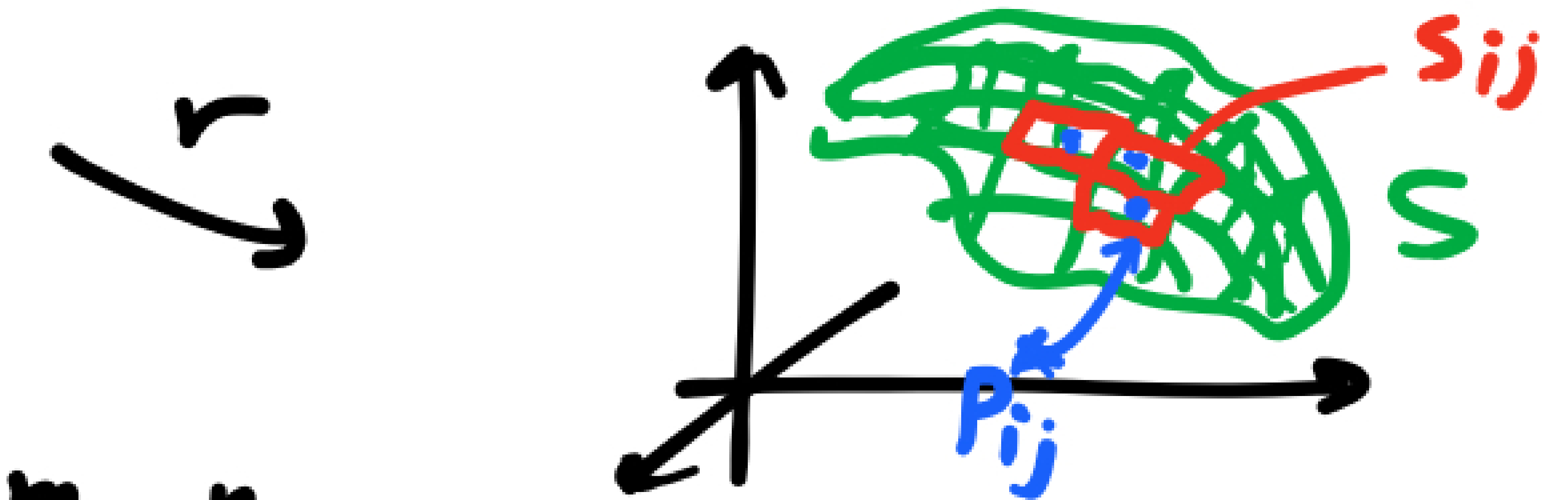
- later we will define another kind of surface integral – surface integral of a vector field.

Construction: • we want to have

$$\iint_S 1 \, dS = \text{surface area}$$



• parametrize S



Riemann sum: $\displaystyle\sum_{j=1}^{m} \sum_{i=1}^{n} f(P_{ij}^{*}) \underbrace{\Delta S_{ij}}_{}$

Define $\displaystyle\iint_S f(x,y,z)\,dS = \lim_{n,m\to\infty} \left(\text{Riemann sum}\right).$

recall that $\Delta S_{ij} \approx \|r_u \times r_v\| \Delta u \Delta v$

We obtain

$$\boxed{\iint_S f(x,y,z)\,dS = \iint_D f(r(u,v))\,\underline{\underline{\|r_u \times r_v\|}}\,dA}$$

<u>Example</u>

$\displaystyle\iint_S x^2+y^2\,dS$, where S - unit sphere centered at the origin.

- $\|r_u \times r_v\| = \sin\varphi$ (from before)
- $f(r(u,v)) = (\sin\varphi\cos\theta)^2 + (\sin\varphi\sin\theta)^2 =$

$$= \sin^2\varphi\cos^2\theta + \sin^2\varphi\sin^2\theta = \sin^2\varphi.$$

$$\iint_S x^2+y^2\,dS = \int_0^{2\pi}\int_0^{\pi} \sin^3\varphi \; d\varphi\, d\theta =$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{\pi} \sin^3\varphi\, d\varphi = 2\pi \cdot \int_0^{\pi} \sin^3\varphi\, d\varphi =$$

$$= 2\pi\left[\frac{\cos^3\varphi}{3} - \cos\varphi\right]_0^{\pi} = \boxed{8\pi/3}$$

$2/3 - (-2/3)$

$\sin^2\varphi\,\sin\varphi = (1-\cos^2\varphi)\sin\varphi =$

$= \sin\varphi - \cos^2\varphi\sin\varphi$