Last time:

- curl - operation on $\underbrace{\text{vector}}_{\text{3d}}$ fields
- $\text{curl}\, F = \nabla \times F$
- $\text{curl}(\nabla f) = \vec{0}$
- thm (converse): F is defined on $\mathbb{R}^3$ and $\text{curl}\, F = \vec{0}$, then F is conservative.

Example

- Show $y^2z^3\,\hat{\imath} + 2xyz^3\,\hat{\jmath} + 3xy^2z^2\,\hat{k} = F$ is conservative

- find f, so that $\nabla f = F$.

Solution:

- F is defined everywhere

$$\operatorname{curl} F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2z^3 & 2xyz^3 & 3xy^2z^2 \end{vmatrix} = \left(\cancel{6xyz^2 - 6xyz^2}\right)\hat{\imath} + \left(\cancel{3y^2z^2 - 3y^2z^2}\right)\hat{\jmath} + \left(\cancel{2yz^3 - 2yz^3}\right)\hat{k}$$

$$= \vec{0}.$$

$= 0.$

By thm, F is conservative.

- $$f_x = y^2z^3$$
  $$f_y = 2xyz^3$$
  $$f_z = 3xy^2z^2 \Rightarrow f = xy^2z^3 + C(x,y)$$

$$\Downarrow$$

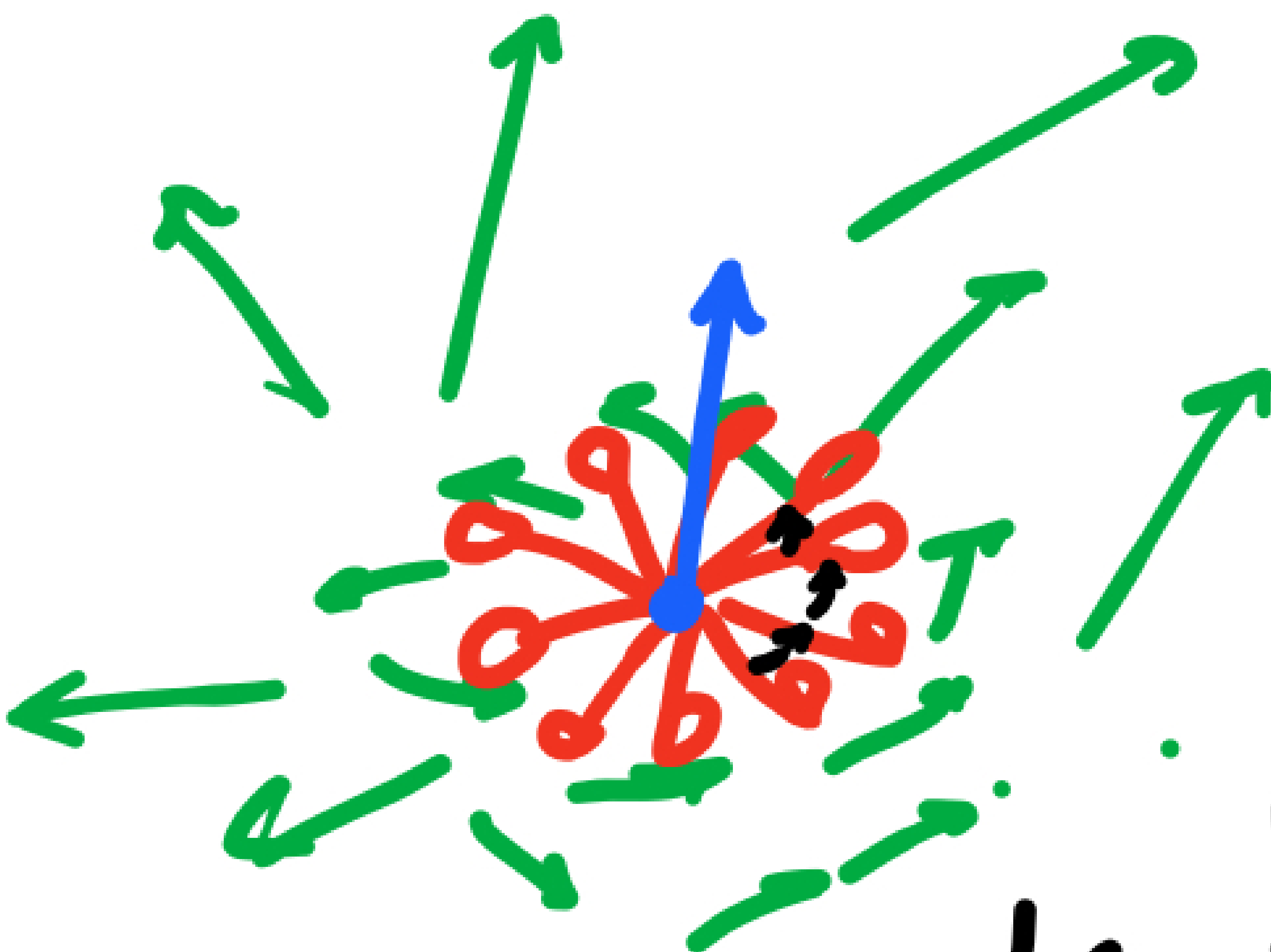
$$C_y(x,y) = 0 \Leftarrow f_y = 2xyz^3 + C_y(x,y)$$

$$\Downarrow$$

$$C(x,y) = g(x) \Rightarrow f_x = y^2z^3 + g'(x)$$

$$\Rightarrow f = xy^2z^3 + C.$$

Geometric meaning of curl F



Sps F is a velocity vector field of a fluid. Put a tiny paddle wheel at point P. The wheel rotates fastest when its axis is parallel to curl F, and magnitude represents speed.

Divergence : operation on vector fields

If $\vec{F} = P\hat{\imath} + Q\hat{\jmath} + R\hat{k}$

is a vector field, so that P, Q and R have partial derivatives

Then $$\boxed{\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}}$$

Symbolically,

$$\boxed{\operatorname{div} F = \nabla \cdot F}$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (P, Q, R) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}.$$

Example $F = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.

$\operatorname{div} F = 1 + 1 + 1 = 3.$

Theorem $\operatorname{div}(\operatorname{curl} F) = 0$

(as a function)

Proof: direct check (exercise).

What does div F measure?