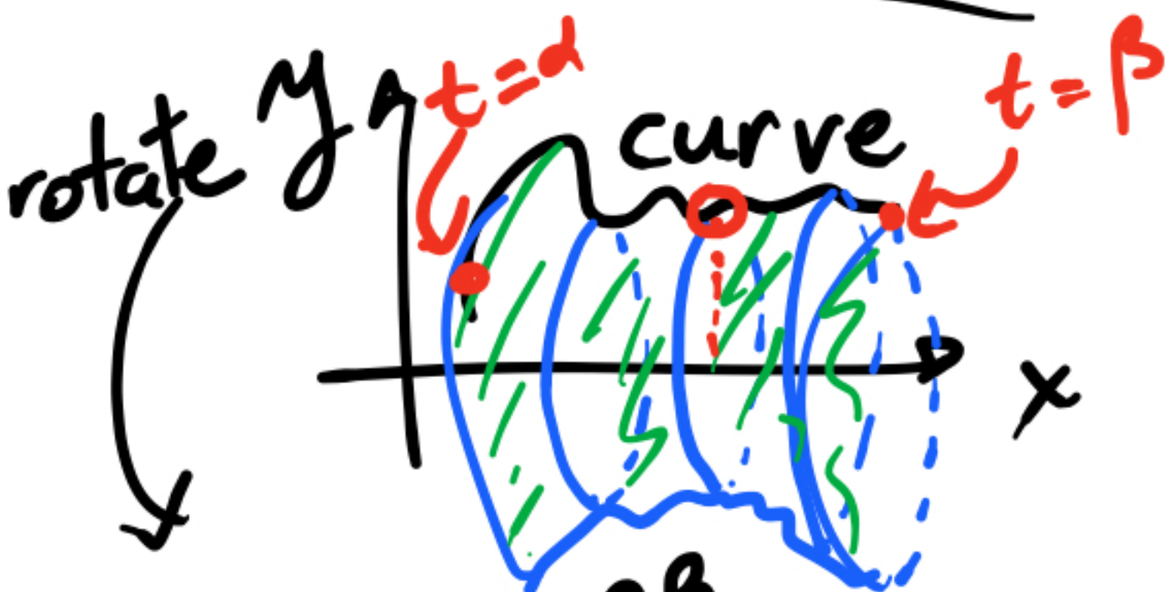
Surface area.

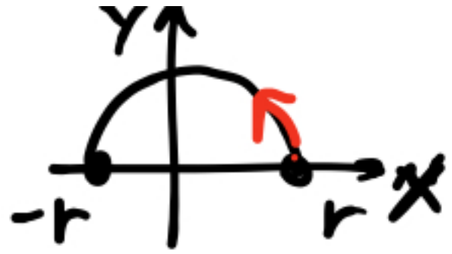


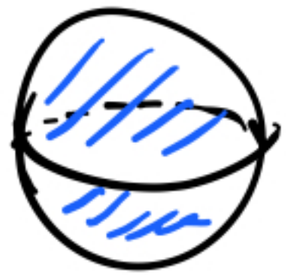
$2\pi y \cdot (S)$

↔ width = arclength

$$S = \int_{\alpha}^{\beta} \underbrace{2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{y\uparrow} \, dt$$

Example





sphere of radius r

$= 4\pi r^2.$

$$\begin{cases} x = r\cos t \\ y = r\sin t \end{cases} \quad 0 \le t \le \pi$$

surface area

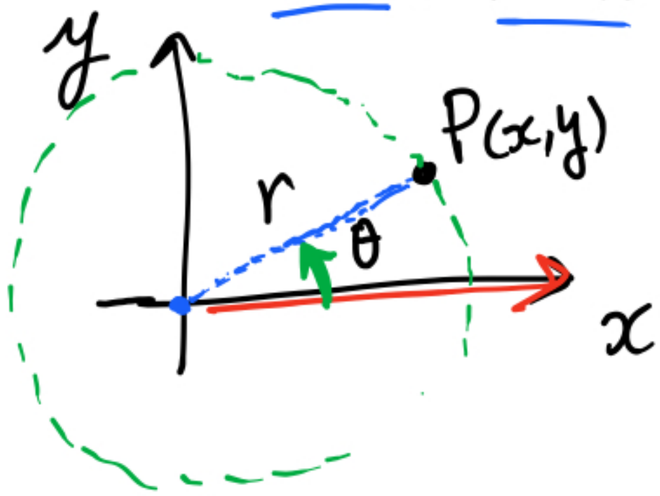
$$S = \int_0^{\pi} 2\pi r \sin t \sqrt{(-r\sin t)^2 + (r\cos t)^2}\, dt =$$

$$= r^2(\sin^2 t + \cos^2 t) = 1$$

$$= \int_0^{\pi} 2\pi r^2 \sin t\, dt = 2\pi r^2 \int_0^{\pi} \sin t\, dt$$

$$= 2\pi r^2 \cdot \underbrace{[-\cos t]\Big|_0^{\pi}}_{=2} = \boxed{4\pi r^2}.$$

10.3. Polar Coordinates



xy-plane
(cartesian coordinates)
(rectangular)

r = distance from P to origin

θ = "polar angle"
angle measured along positive x-axis
in a counterclockwise manner.

Polar coordinates for P are (r, θ)

- polar coordinates allow to describe objects that are circular in nature with simpler equations.