Vector Fields (2D)

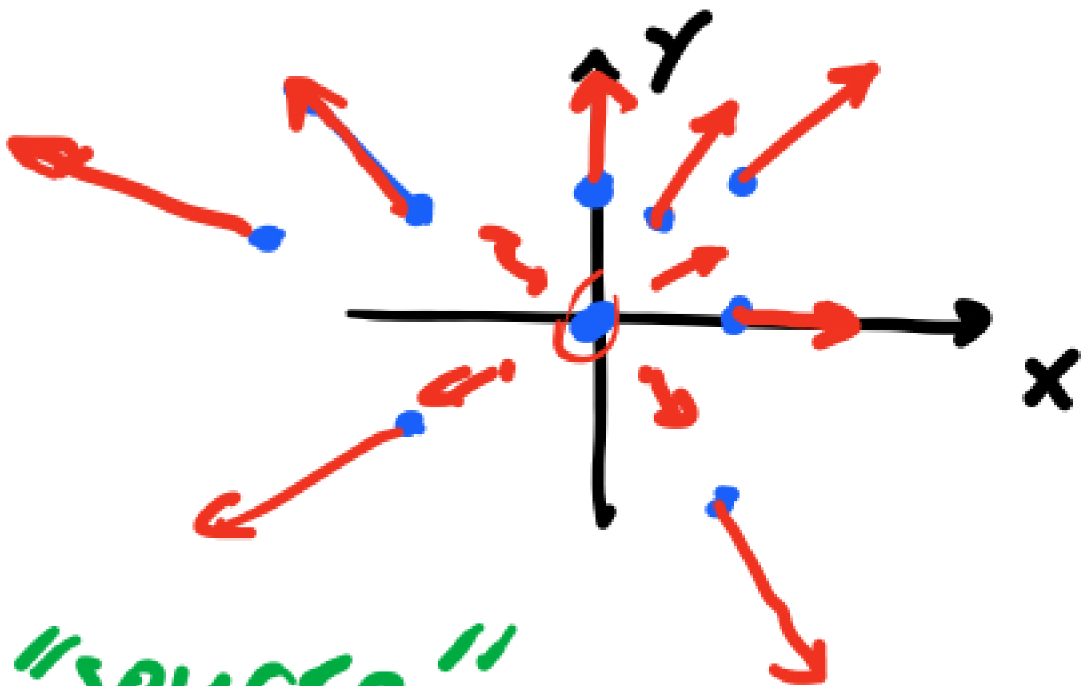
Def. A vector field on a domain $D \subset \mathbb{R}^2$ is a function F that assigns to a point (x,y) in D a two-dimensional vector $F(x,y)$.

$F(x,y)$ can be written as follows

$$F(x,y) = P(x,y)\,\hat{i} + Q(x,y)\,\hat{j} = \langle P, Q \rangle$$

component functions
(regular functions of 2 var.)

Example Sketch $F(x,y) = x\hat{\imath} + y\hat{\jmath} = \langle x,y\rangle$



"sourse"

$F(0,0) = \vec{0}$

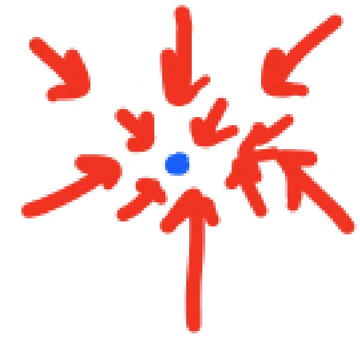
$F(1,0) = \hat{\imath}$

$F(0,1) = \hat{\jmath}$

$F(1,1) = \hat{\imath} + \hat{\jmath}$

$\langle -x, -y \rangle$

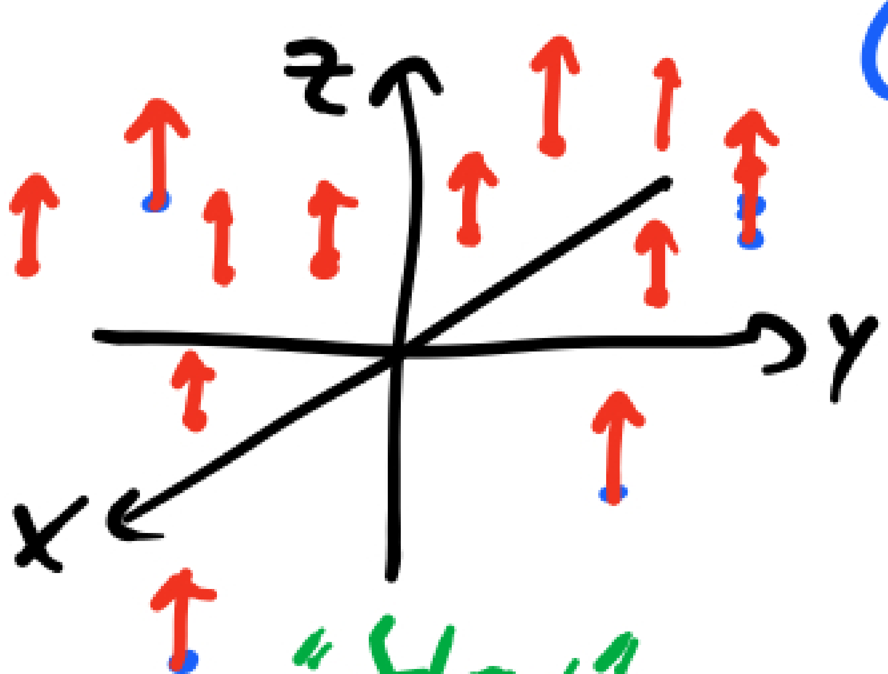
"sink"



3D vector fields defined similarly.

e.g. $F(x,y,z) = \hat{k} = \langle 0, 0, 1 \rangle$

(or component functions are $0, 0, 1$)



"flow"

- We say a vector field is continuous if the component functions are continuous

Gradient vector field

$$f(x,y) \xmapsto{\nabla} \nabla f \text{ (grad } f)$$

(differentiable) function $\qquad$ gradient vector field

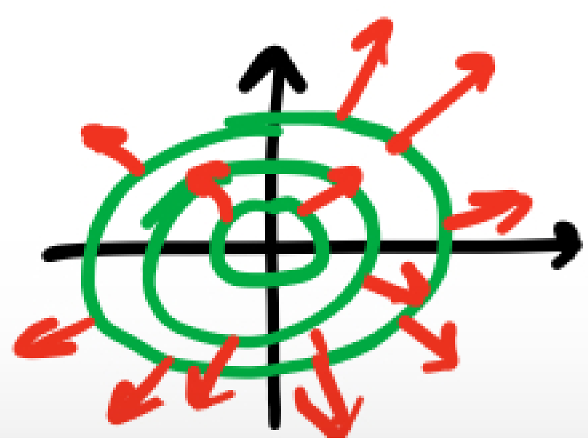
$$\nabla f(x,y) = f_x(x,y)\,\hat{i} + f_y\,\hat{j}.$$

Example $f(x,y) = \dfrac{x^2+y^2}{2}$

$$\nabla f = x\,\hat{i} + y\,\hat{j}$$

level curves of $f(x,y)$:

$$\frac{x^2+y^2}{2} = k$$



Example?

"rotational" behavior

$$F(x,y) = y\hat{i} - x\hat{j} = \langle y, -x \rangle$$

~~tha~~ Is it a gradient vector field of some function?

$$(y,-x)\cdot(x,y) = xy - xy = 0.$$

$$f(x,y) = \frac{y^2 - x^2}{2}$$

$$\nabla f = -x\hat{i} + y\hat{j}. \quad ✗$$

- Maybe it's not a gradient vector field of any function?

(answer: next week!)